Lesson Overview

LESSON 33
Analyze Patterns and Relationships

CCSS Focus

Domain
Operations and Algebraic Thinking

Cluster
B. Analyze patterns and relationships.

Standard
5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Additional Standard
5.G.A.2 (See Standards Correlations at the end of the book for full text.)

Lesson Objectives

Content Objectives
• Generate a numerical pattern given a rule.
• Identify relationships between corresponding terms of two patterns.
• Graph corresponding terms of two patterns as ordered pairs in the first quadrant of the coordinate plane.

Language Objectives
• List a sequence of numbers based on a given verbal rule.
• Identify corresponding terms of two numerical patterns.
• Describe the mathematical relationship between corresponding terms of two number patterns.
• Locate in the coordinate plane the point named by an ordered pair formed from the corresponding terms of two patterns.

Prerequisite Skills
• Use addition, subtraction, multiplication, and division.
• Recognize and extend number patterns.
• Graph ordered pairs in the coordinate plane.

Lesson Vocabulary
• corresponding terms terms that have the same position in two related patterns. For example, the second term in one pattern and the second term in a related pattern are corresponding terms.
• terms the numbers or shapes in a pattern.

Learning Progression

In Grade 4 students analyzed number and shape patterns. They described and generated number patterns and identified explicit rules in patterns. In the previous lessons, Grade 5 students were introduced to the coordinate plane and learned to graph ordered pairs in the first quadrant.

In this lesson students continue their work with number patterns as they begin to look at the relationships between patterns. Students analyze numerical patterns and describe the relationship between corresponding terms of two different numerical patterns. Students graph ordered pairs of corresponding terms in the first quadrant of the coordinate plane to help them see the relationship between corresponding terms.

In Grade 6 students will build on their understanding of the relationship between corresponding terms of two related patterns when they work with tables and graphs of equivalent ratios and begin to reason about relationships between an independent variable and a dependent variable.

*See page 305m to see how every lesson includes these SMPs.
## Lesson Pacing Guide

### Whole Class Instruction

<table>
<thead>
<tr>
<th>SESSION 1</th>
<th>Explore</th>
<th>45–60 min</th>
</tr>
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<tbody>
<tr>
<td><strong>Session</strong></td>
<td><strong>Analyze Patterns and Relationships</strong></td>
<td><strong>Additional Practice</strong></td>
</tr>
<tr>
<td>Start</td>
<td>5 min</td>
<td>Lesson pages 683–684</td>
</tr>
<tr>
<td>Try It</td>
<td>10 min</td>
<td></td>
</tr>
<tr>
<td>Discuss It</td>
<td>10 min</td>
<td></td>
</tr>
<tr>
<td>Connect It</td>
<td>15 min</td>
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<tr>
<td>Close: Exit Ticket</td>
<td>5 min</td>
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<table>
<thead>
<tr>
<th>SESSION 2</th>
<th>Develop</th>
<th>45–60 min</th>
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<tbody>
<tr>
<td><strong>Session</strong></td>
<td><strong>Comparing Two Numerical Patterns</strong></td>
<td><strong>Additional Practice</strong></td>
</tr>
<tr>
<td>Start</td>
<td>5 min</td>
<td>Lesson pages 689–690</td>
</tr>
<tr>
<td>Try It</td>
<td>5 min</td>
<td></td>
</tr>
<tr>
<td>Discuss It</td>
<td>10 min</td>
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<tr>
<td>Picture It &amp; Model It</td>
<td>5 min</td>
<td></td>
</tr>
<tr>
<td>Connect It</td>
<td>15 min</td>
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</tr>
<tr>
<td>Close: Exit Ticket</td>
<td>5 min</td>
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<table>
<thead>
<tr>
<th>SESSION 3</th>
<th>Develop</th>
<th>45–60 min</th>
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<tbody>
<tr>
<td><strong>Session</strong></td>
<td><strong>Using a Graph to Compare Patterns</strong></td>
<td><strong>Additional Practice</strong></td>
</tr>
<tr>
<td>Start</td>
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<td>Lesson pages 695–696</td>
</tr>
<tr>
<td>Try It</td>
<td>5 min</td>
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</tr>
<tr>
<td>Discuss It</td>
<td>10 min</td>
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<tr>
<td>Model Its</td>
<td>5 min</td>
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</tr>
<tr>
<td>Connect It</td>
<td>15 min</td>
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<tr>
<td>Close: Exit Ticket</td>
<td>5 min</td>
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<table>
<thead>
<tr>
<th>SESSION 4</th>
<th>Refine</th>
<th>45–60 min</th>
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<tbody>
<tr>
<td><strong>Session</strong></td>
<td><strong>Analyze Patterns and Relationships</strong></td>
<td><strong>Lesson Quiz</strong></td>
</tr>
<tr>
<td>Start</td>
<td>5 min</td>
<td></td>
</tr>
<tr>
<td>Example &amp; Problems 1–3</td>
<td>15 min</td>
<td></td>
</tr>
<tr>
<td>Practice &amp; Small Group Differentiation</td>
<td>20 min</td>
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<tr>
<td>Close: Exit Ticket</td>
<td>5 min</td>
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</tbody>
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### Small Group Differentiation

**PREPARE**

**Ready Prerequisite Lesson**
- **Grade 4**
  - Lesson 9 Number and Shape Patterns

**RETEACH**

**Tools for Instruction**
- **Grade 4**
  - Lesson 9 Numeric Patterns
- **Grade 5**
  - Lesson 33 Numerical Patterns and Relationships

**REINFORCE**

**Math Center Activities**
- **Grade 5**
  - Lesson 33 Use Number Sequence Vocabulary
  - Lesson 33 Plot Points

**EXTEND**

**Enrichment Activity**
- **Grade 5**
  - Lesson 33 Class Fundraiser

### Independent Learning

**PERSONALIZE**

**i-Ready Lessons**
- **Grade 5**
  - Analyze Patterns and Relationships
  - Practice: Analyze Patterns and Relationships

### Lesson Materials

**Lesson** *(Required)*
- **Per student:** base-ten blocks (30 ones units)

**Activities**
- **Per pair:** counters (45 in one color and 40 in a second color)
- **For display:** empty table and coordinate plane similar to those in the Session 3 Model Its

**Math Toolkit**
- counters, base-ten blocks, grid paper

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*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.*
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

**Connect to Family**

Use the Family Letter—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

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**Activity: Analyzing Patterns**

Do this activity with your child to analyze patterns and relationships.

- Together with your child, find the cost of your child’s two favorite snacks. Round each to the nearest dollar. (Example: A box of crackers costs $2 and a carton of ice cream costs $3.)
- In the table, write the cost of 0, 1, 2, 3, 4, and 5 containers of each snack.

<table>
<thead>
<tr>
<th>Snack 1</th>
<th>Snack 2</th>
<th>Ordered Pair (x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

A diagram shows the number patterns for the raisins and the nuts:

- At the school fair, a box of raisins costs $2 and a box of nuts costs $4. How does the cost of a given number of boxes of raisins compare to the cost of the same number of boxes of nuts for 0, 2, 4, 6, or 8 boxes?

You can list the numbers, or terms, of the pattern in a table and form ordered pairs of corresponding terms.

- Look for a pattern. The second number in each ordered pair is twice the first number. For example, in the ordered pair (4, 8), 8 = 2 x 4.

Another way to see how the number patterns are related is to plot the ordered pairs on a graph.

The graph at the right shows a point for each ordered pair in the table. From point to point, the pattern is: move 2 to the right, move up 4.

- Invite your child to share what he or she knows about analyzing number patterns and relationships by doing the following activity together.

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**Goal**

The goal of the Family Letter is to provide students and family members with opportunities to explore how to analyze patterns and relationships.

**Activity**

In the Analyzing Patterns activity, students and family members use tables and graphs to analyze patterns and relationships. Provide photos of sample snacks and prices to aid students in completing the activity at home.

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**Math Talk at Home**

Encourage students to talk with their family members about analyzing patterns and relationships. Have students provide two number patterns for their family members to analyze, such as 1, 2, 3, 4, 5 and 10, 20, 30, 40, 50.

**Conversation Starters** Below are additional conversation starters students can write in their Family Letter or math journal to engage family members in conversations about number patterns and relationships.

- How is the number of items you purchase at a grocery store related to their total cost?
- What other number patterns or relationships do you see at work or at home?
**Connect to Community and Cultural Responsiveness**

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

**Sessions 1–4 Use throughout the lesson.**

- Point out that two number patterns can be used in daily activities to show how costs are related. For example, if you know the price of an admission ticket to a museum is $5, you can calculate the total cost for any number of people visiting the museum. The two number patterns are the pattern for the number of people, 1, 2, 3, 4, 5, . . ., and the pattern for the total cost, $5, $10, $15, $20, $25, . . . .

- Encourage students to think of situations in which it is useful to analyze the relationship between two number patterns. Call on volunteers to provide examples. Have students suggest word problems with number patterns for the class to solve.

- Remind students that organizing numbers into patterns can help simplify a problem. Review how to use a table and a graph to organize numbers in problems. Encourage students to think about how they organize different things in their homes or at school and why the organization is useful. Call on volunteers to provide examples.

**Connect to Language Development**

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

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**ELL English Language Learners: Differentiated Instruction**

<table>
<thead>
<tr>
<th>Levels 1–3</th>
<th>Levels 2–4</th>
<th>Levels 3–5</th>
</tr>
</thead>
</table>

**Speaking/Writing** Read the Try It problem and have students form pairs. Ask the following questions and allow time for pairs to talk about their answers: What two types of snacks does Maria sell? What is the cost of each type of snack? Have students work with their partner to say numbers that are multiples of 3 and 6. Then have partners write the lists of costs. Provide the following sentence frames for students to complete in writing:

- The cost of _______ sandwiches is $ _______.
- The cost of _______ frozen yogurts is $ _______.
- To find the cost of frozen yogurt, for each yogurt _______.
- To find the cost of sandwiches, for each sandwich _______.

**Speaking/Writing** Read the Try It problem and have students form pairs. Ask students to work with their partner and restate the information using the following sentence frames:

- Maria sells _______ and _______ at the snack bar.
- Each _______ costs _______ and each _______ costs _______.

Have partners say numbers that are multiples of 3 and 6. Then have partners write the lists of costs. Ask students to write a rule for each using the following sentence frame:

- For the frozen yogurt, the rule is _______.
- For the sandwiches, the rule is _______.

**Speaking/Writing** Have students form pairs and read the Try It problem. Ask students to discuss with their partner what they need to find. Encourage students to restate the problem in their own words using complete sentences.

Have partners discuss what words they can use to describe the patterns. Ask students to write a rule for the cost of the sandwiches and the cost of the yogurts using complete sentences. Call on students to read their sentences and explain the model or strategy they used to determine the rule.
**Purpose** In this session students draw on their Grade 4 experience with patterns. They will share models to explore generating two numerical patterns. They will look ahead to think about rules used to generate number patterns and identify corresponding terms of two number patterns.

**Connect to Prior Knowledge**

**Materials** For each student: base-ten blocks (30 ones units)

**Why** Review extending and describing number patterns to prepare for generating and comparing two related number patterns.

**How** Have students extend three number patterns. Provide base-ten blocks to help. Have partners discuss how the patterns are alike and different.

**Start**

**Solutions**

Write the next three numbers in each pattern.

- 0, 4, 8, 12, 16, 
- 8, 4, 0
- 2, 8, 14, 20, 26, 
- 28, 24, 20, 16, 12

**Listen for** Numbers in the 1st and 2nd patterns increase. Numbers in the 1st and 3rd patterns change by 4.

**TRY IT**

**Make Sense of the Problem**

To support students in making sense of the problem, have them identify that this problem involves making two lists of costs and explaining how to find each consecutive cost.

**DISCUSS IT**

**Support Partner Discussion**

Encourage students to discuss how they organized their work to keep track of the costs of different numbers of items. Look for, and prompt as necessary for, understanding of:

- the cost of each item
- the need to track the cost of 0–6 of each item
- how the cost changes with each additional item

**Common Misconception** Look for students who try to combine the two lists. As students present solutions, have them write a label for each pattern.

**Select and Sequence Student Solutions**

One possible order for whole class discussion:

- concrete models used to find costs
- diagrams or drawings used to list costs
- tables used to list costs

**Support Whole Class Discussion**

Prompt students to note different ways to write the lists and find the rules.

**Ask** How do [student name]’s and [student name]’s models show the costs of each number of each food item and the rule for each pattern?

**Listen for** Lists of costs may be shown in tables that also show the number of items or in lists that show just the costs. The rules are shown in the list by the number used to skip-count in each list. The table rules are shown by how much each cost changes from one column to the next.
**Lessons 33**

**Analyze Patterns and Relationships**

**Exploring Patterns and Relationships**

**Session 1**

**Look Back**

How did you find the costs of yogurts and the costs of sandwiches?

Possible answer: I made tables showing the costs for yogurts and the costs for sandwiches. To find the costs for yogurts, I added $3 for each yogurt. To find the costs for sandwiches, I added $6 for each sandwich.

**Look Ahead**

In the previous problem, each list of costs forms a numerical pattern. The numbers in a pattern are called terms. A rule tells you how to move from one term in a pattern to the next. You can generate two related patterns using two different rules.

a. Maria also sells hot dogs for $5 each and pizzas for $10 each. Use the rules shown in the table to list the terms of two patterns.

<table>
<thead>
<tr>
<th>Number of Items</th>
<th>Cost of Hot Dogs ($)</th>
<th>Cost of Pizzas ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>30</td>
</tr>
</tbody>
</table>

b. Terms that are in the same position in two related patterns are called corresponding terms. What are the corresponding terms for the cost of 3 items in the two patterns you wrote in the table in part a?

15 and 30

**Reflect**

How do the corresponding terms in problem 2b compare? Is this true for all pairs of corresponding terms in the table?

Possible answer: The term for the cost of hot dogs is half the value of the term for the cost of pizzas. This is true for all of the corresponding terms in the table.

**Close: Exit Ticket**

**Reflect**

Look for understanding that all corresponding terms in two number patterns have the same relationship.

**Common Misconception**

If students describe the comparison in problem 2b as “add 15,” then have them look at the relationship between corresponding terms for 1 and 2 of each item purchased. Tell them to think about the relationship as one quantity being a number of times the other quantity. Ask if this “times as many” relationship is the same for 1, 2, and 3 items bought.

**Real-World Connection**

Encourage students to think about everyday places or situations in which numerical patterns may occur and how comparing the patterns is useful. Have volunteers share their ideas. Examples: cost of a pound of apples compared to the cost of a pound of watermelons, cost of a number of boxes of pencils compared to the cost of a number of boxes of markers.

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**Hands-On Activity**

Use counters to generate patterns.

**If . . . students are unsure about the concept of finding a rule for number patterns,**

**Then . . . use this activity to have them find a rule for patterns with counters.**

**Materials**

For each pair: counters (40 in each of 2 colors)

- Tell students that Maria also lists the costs of water ($2/bottle) and trail mix ($4/bag).
- Tell students to use one color of counters to represent the cost of bottles of water and the other color to represent the cost of bags of trail mix. Have students show the cost of 1, 2, 3, and 4 of each item.
- Ask: How did you find the cost of 2 water bottles? [I added two sets of 2 counters.]
- How did you find the cost of 3 water bottles? [I added one more set of 2 counters.]
- What is the rule for the cost of water bottles? [Add 2]
- Ask: How did you find the cost of 2 bags of trail mix? [I made two sets of 4 counters.]
- How did you find the cost of 3 bags of trail mix? [I added one more set of 4 counters.]
- What is the rule for the cost of bags of trail mix? [Add 4]

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**Connect It**

1. **Look Back**

Look for understanding that you can use organized lists (e.g., tables) to show the costs of different numbers of an item.

2. **Look Ahead**

Point out that when students listed the costs of yogurts or sandwiches, they made a number pattern and that each number in that pattern is called a term. Discuss how corresponding terms in two number patterns share the same place in the pattern, such as the costs of 3 yogurts and 3 sandwiches. Students will learn more about corresponding terms in the Additional Practice.

Students should be able to complete the terms and corresponding terms for the costs of hot dogs and costs of pizza patterns.
Prepare for Analyzing Patterns and Relationships

1. Think about what you know about patterns. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

   Possible answers:

   **What Is It?**
   - the numbers that have the same position in two related patterns

   **What I Know About It**
   - The second term in one pattern and the second term in a related pattern are corresponding terms.

   Examples

<table>
<thead>
<tr>
<th>Pattern 1</th>
<th>Pattern 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 3, 6, 9, 12</td>
<td>0, 1, 2, 3, 4</td>
</tr>
<tr>
<td>+3</td>
<td>+1</td>
</tr>
<tr>
<td>0, 3, 6, 9, 12</td>
<td>0, 1, 2, 3, 4</td>
</tr>
<tr>
<td>+3</td>
<td>+1</td>
</tr>
<tr>
<td>0, 3, 6, 9, 12</td>
<td>0, 1, 2, 3, 4</td>
</tr>
<tr>
<td>+3</td>
<td>+1</td>
</tr>
<tr>
<td>0, 3, 6, 9, 12</td>
<td>0, 1, 2, 3, 4</td>
</tr>
<tr>
<td>+3</td>
<td>+1</td>
</tr>
</tbody>
</table>

   Possible answer: The corresponding terms are the terms in each pattern that are in the same place. The first corresponding terms are 0 and 0. The second are 3 and 1. The rest are 6 and 2, 9 and 3, and 12 and 4.

Support Vocabulary Development

1. Ask students to look at the graphic organizer and read the term in the middle. Tell students that the word corresponding refers to things that are compared. Point out that comparisons are usually made between things that have something in common, such as the position in a pattern. Invite students to share different meanings for the word terms. Remind students that a term in a numerical pattern is one of the numbers.

   Have students discuss with a partner how to complete the boxes for What Is It? and What I Know About It. Ask pairs to write words and phrases that represent their thinking. Have students scan the pages of the session for different ways two numerical patterns can be displayed. Encourage students to use examples from the session or come up with their own ideas. Call on volunteers to share their examples.

2. Have students work with a partner to write a rule for each pattern. Have students use two fingers to point to corresponding terms in the patterns. Invite students to share their rules before identifying corresponding terms.

Supplemental Math Vocabulary

- compare
3. Assign problem 3 to provide another look at analyzing patterns and relationships.

This problem is similar to the problem about Maria working at the snack stand. In both problems, students are given a word problem that requires listing two number patterns and identifying the rule for each pattern.

Students may want to use counters, base-ten blocks, or grid paper.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- What is this problem about?
- What is the question I am trying to answer?
- What information is important?

**Solution:**

T-shirt costs: $0, $15, $30, $45, $60, $75, $90; snow globe costs: $0, $5, $10, $15, $20, $25, $30;
The rule for the cost of T-shirts is add 15 and the rule for the cost of snow globes is add 5.

**Medium**

4. Have students solve the problem a different way to check their answer.
LESSON 33  Develop

**Purpose** In this session students solve a problem that requires comparing two numerical patterns. Students model the patterns either on paper or with manipulatives to describe how terms in the patterns relate. The purpose of this problem is to have students develop strategies for comparing terms in two numerical patterns and to introduce them to using ordered pair notation for writing pairs of corresponding terms.

**Start**

**Connect to Prior Knowledge**

**Why** Describe relationships between numbers to prepare for comparing numerical patterns.

**How** Have partners describe at least two relationships they see between terms, either within one pattern or between the patterns.

Possible Solutions

<table>
<thead>
<tr>
<th>Monsters Caught</th>
<th>Level 1 Points</th>
<th>Ghosts Caught</th>
<th>Level 2 Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>6</td>
<td>48</td>
</tr>
</tbody>
</table>

The points earned for catching ghosts in Level 2 are 4 times the points earned for catching monsters in Level 1.

Possible student work:

**Sample A**

<table>
<thead>
<tr>
<th>Monsters Caught</th>
<th>Level 1 Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
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<tr>
<td>4</td>
<td>8</td>
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<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

**Sample B**

<table>
<thead>
<tr>
<th>Ghosts Caught</th>
<th>Level 2 Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>48</td>
</tr>
</tbody>
</table>

**Develop Language**

**Why** Reinforce the meaning of compare.

**How** Write the word compare on the board. Remind students that compare means to determine how two or more things are alike and different. Invite students to provide types of numerical patterns (e.g., increasing by a factor of 2, decreasing by 1). Have students compare the patterns by telling which patterns are similar or different.

**TRY IT**

**Make Sense of the Problem**

To support students in making sense of the problem, have them identify the number of points per catch at each level and what the problem asks them to compare.

**Ask** How many points do you get for catching a monster? a ghost? What are you supposed to compare?

**DISCUSS IT**

**Support Partner Discussion**

Encourage students to compare how they organized the number of points at each level as they discuss their solutions.

Support as needed with questions such as:

- How did you get started?
- How did you decide how to keep track of points at each level? Why?

**Common Misconception** Look for students who identify when points match in Level 1 and Level 2, such as 8 points for 4 monsters and 8 points for 1 ghost, instead of comparing points for each number of monsters or ghosts caught.

**Select and Sequence Student Solutions**

One possible order for whole class discussion:

- concrete models used to find and compare points
- organized drawings used to list and compare points
- tables used to list and compare points
- lists that show evidence of using the rules “add 2” and “add 8”
- solutions that include the list 0, 1, 2, 3, 4, 5, 6 as well as lists of points
Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask  Where does your model show the number of Level 1 points? the number of Level 2 points? How does your model help you compare the Level 2 points to the Level 1 points?

Listen for  Students should recognize that accurate responses include Level 1 points that increase by 2 for each monster caught and Level 2 points that increase by 8 for each ghost caught. Responses should include that the number of Level 2 points is always 4 times the number of Level 1 points for the same number of creatures caught.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:
- points for catching monsters (Level 1)
- points for catching ghosts (Level 2)
- Level 1 and 2 number patterns

Ask  How do the diagram and the table each show the number of points for different numbers of monsters and ghosts caught?

Listen for  The diagram shows the points for monsters and ghosts in a row, adding 2 or 8 points for each additional monster or ghost caught. The table shows the same information in columns without labeling the +2 and +8 changes.

For the diagram, prompt students to consider the patterns shown by the lists of points.
- What is the rule for the Level 1 pattern?
- What is the rule for the Level 2 pattern?
- What would be the next number in each pattern? What would that number mean?

For the table model, prompt students to discuss lesson vocabulary: term and corresponding terms.
- What are the terms in the Level 1 and 2 patterns?
- What is an example of corresponding terms in the table?

Deepen Understanding

Table Model

SMP 7  Look for structure.

When discussing the table model, prompt students to consider how the table was constructed.

Ask  Why does it make sense that 2 points and 8 points are listed in the same row of the table? Why are 4 points and 16 points listed in the next row?

Listen for  2 points and 8 points are the points for 1 monster and 1 ghost caught. 4 points and 16 points are the points for an additional monster or ghost caught, or a total of 2 monsters or 2 ghosts caught.

Ask  What is the benefit of listing the points in order of number of monsters or ghosts caught? How does this order help you find the numbers for the next row in the table?

Listen for  By listing the points in order, you can see a number pattern in each column. For Level 1, you can just add 2 to find the number in the next row. For Level 2, you can add 8 to find the number in the next row.
LESSON 33
SESSION 2 Develop

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page, students will describe different patterns and relationships within a level or between levels of the game.

Monitor and Confirm

1 – 2 Check for understanding that:
- the Level 1 rule is add 2
- the Level 2 rule is add 8
- the corresponding terms are in the same row in the table

Support Whole Class Discussion

3 Tell students that this problem will prepare them to provide the explanation required in problem 4.

Ask  What is a multiplication equation that shows the relationship between the corresponding terms? If you wrote a similar equation for each pair of corresponding terms, which factor would always stay the same? Why?

Listen for  2 × 4 = 8; The factor 4 would always stay the same because the second number is always 4 times the first number in each pair of corresponding terms.

4 Look for the idea that you could show columns in the table for Level 1 and Level 3 points, and compare the corresponding terms from those columns.

5 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to identify relationships between two numerical patterns.

1 Look at Picture It on the previous page. Describe how the total number of points changes with each monster or ghost caught in Level 1 and Level 2.
   - Level 1 rule: Each number increases by 2 (add 2).
   - Level 2 rule: Each number increases by 8 (add 8).

2 Complete the table on the previous page. See table.

3 Use the table to describe the relationship between the corresponding terms of the patterns.
   - Possible answer: Each row contains the corresponding terms of the pattern. The number of points for Level 2 in each row is four times the number of points for Level 1 in the same row. The term of the Level 2 pattern is four times the corresponding term of the Level 1 pattern.

4 Suppose the game has a third level. You get 10 points for each giant caught in Level 3. Explain how you could use the table to show how the points for catching a given number of monsters in Level 1 compares to catching the same number of giants in Level 3.
   - Possible answer: You could put another column in the table for Level 3 that shows 10 points for each giant caught. Then you could compare Level 1 points as the first number and Level 3 points as the second number. Look for relationships in the corresponding terms. Level 3 points are five times Level 1 points for the same number of giants or monsters caught.

5 REFLECT

Look back at your Try It, strategies by classmates, and Picture It and Model It. Which models or strategies do you like best for representing patterns and identifying relationships between corresponding terms? Explain.

Some students may say they like using a table because it is easy to compare the corresponding terms when they are side-by-side.

Visual Model

Help students relate diagrams of numerical patterns to tables.

If . . . students do not recognize the patterns in the table model,
Then . . . have them use this activity to label the patterns in the table.

- Have students look at the table in Model It. Tell students that they can label the columns in their books in the same manner as you will show at the board.
- Copy the Level 1 column of the table on the board. Discuss that the number of points increases by 2 for each monster caught.
- Draw and label arrows with “+ 2” from one row to the next. Ask students to compare the first diagram in Picture It to the first column of the table as you have labeled it. [The numbers are the same, and the changes are the same. The only thing not the same is that one list is horizontal and one is vertical.]
- Copy the Level 2 column on the board. Ask a volunteer to label the Level 2 column in a similar manner. [arrows with “+ 8” from each row to the next]
- Ask for a volunteer to make a column for Level 3 points based on problem 4. Have students identify and label the change in points from row to row. [+10]
APPLY IT
For all problems, encourage students to use diagrams or tables to help organize patterns.

6 School magnets cost $4, and shirts cost $24. Write a pattern for the costs of 0–5 magnets and a second pattern for the costs of 0–5 shirts. How do the corresponding terms of the two patterns compare?

Solution magnets ($) 0, 4, 8, 12, 16, 20; shirts ($) 0, 24, 48, 72, 96, 120; Each term in the pattern of shirt costs is six times the corresponding term in the pattern of magnet costs. Students may also say that each term in the pattern of magnet costs is 1/6 of the corresponding term in the pattern of shirt costs. Each term in the pattern of shirt costs is six times the corresponding term in the pattern of magnet costs. Students may also make a table or write ordered pairs in the form (cost of magnets, cost of shirts) or in the form (cost of shirts, cost of magnets).

7 Tom and Ehrin write number patterns. Tom uses the rule “add 3” and starts at 12. Ehrin uses the rule “subtract 4” and starts at 26. Write the first five terms of their patterns. What number appears as a term in both patterns? Show your work.

Possible student work:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tom’s Pattern</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>24</td>
</tr>
<tr>
<td>Ehrin’s Pattern</td>
<td>26</td>
<td>22</td>
<td>18</td>
<td>14</td>
<td>10</td>
</tr>
</tbody>
</table>

Solution 18

8 The Lakeview Feed Store posts prices for two different types of grass seed by the square foot.

<table>
<thead>
<tr>
<th>Number of Square Feet</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grass Seed A ($)</td>
<td>0</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Grass Seed B ($)</td>
<td>0</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
</tr>
</tbody>
</table>

Which sentences correctly compare the cost of a number of square feet of Grass Seed A and the cost of the same number of square feet of Grass Seed B?

A The cost of Grass Seed A is one-half the cost of Grass Seed B.
B The cost of Grass Seed A is two times the cost of Grass Seed B.
C The cost of Grass Seed B is one-half the cost of Grass Seed A.
D The cost of Grass Seed B is two times the cost of Grass Seed A.
E The cost of Grass Seed A is always two dollars less than the cost of Grass Seed B.

Error Alert If students choose B or C, then have them model each pattern with a drawing or counters to understand the comparison. Ask them, for 1 square foot, whether $4 (Grass Seed A cost) is two times $8 (Grass Seed B cost) or $8 (Grass Seed B cost) is two times $4 (Grass Seed A cost).
LESSON 33  SESSION 2 Additional Practice

Practice Comparing Two Numerical Patterns

Study the Example showing one way to identify relationships between two numerical patterns. Then solve problems 1–6.

**EXAMPLE**

The school store sells laces and decals in the school colors. Laces cost $1 each, and decals cost $5 each. Find the cost of laces and the cost of decals for selling 0, 1, 2, 3, 4, and 5 of each item.

Use a table to show the two patterns.

<table>
<thead>
<tr>
<th>Number of Items</th>
<th>Cost of Laces ($)</th>
<th>Cost of Decals ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>

1. Look at the Example. What is the cost for 6 decals? Explain how you got your answer.

   $30; Students’ explanations may indicate that they continued the pattern of “add 5,” or multiplied 6 by 5.

2. Look at the Example. How are the terms in the pattern for the cost of decals related to the corresponding terms in the pattern for the cost of laces?

   Answers will vary. Possible answer: The terms in the decals pattern are 5 times the corresponding terms in the laces pattern.

**Solutions**

1. $30; See possible explanation on the student page.
   *Basic*

2. The terms in the decals pattern are five times the corresponding terms in the laces pattern. Students may write equations to support their explanations.
   *Medium*

**Fluency & Skills Practice**

 Assign Comparing Two Numerical Patterns

In this activity students compare similar and related numerical patterns. This activity encourages students to look closely at patterns to determine and articulate their relationships and differences. This skill is useful in everyday life, such as when finding the better deal between two similarly priced goods.
3. See the completed table on the student page. 

**Challenge**

4. The terms in the pattern for the cost of the bookmarks are three times the corresponding terms in the pattern for the cost of laces. Students may use drawings or equations to support their thinking. 

**Challenge**

5. Add 3; Students may draw arrows and use labels to note the change from row to row for the cost of the bookmarks.  

**Medium**

6. B; 30 is three times 10.  
   C; 36 is three times 12.  
   F; 60 is three times 20.  

**Medium**

---

**Suppose school bookmarks cost $3 each. Complete the table to show how the terms in this pattern compare to the corresponding terms in the pattern for the cost of laces.**

<table>
<thead>
<tr>
<th>Number of Items</th>
<th>Cost of Laces ($)</th>
<th>Cost of Bookmarks ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

**Look at problem 3. How do the corresponding terms of the two patterns compare?**

**Answers will vary. Possible answer:** The terms in the second pattern are three times the corresponding terms in the first pattern.

**Look at problem 3. What is the rule for finding the cost of bookmarks?**

**Add 3.**

**Look at problem 3. If the table was continued, which two values could be corresponding terms for laces and bookmarks?**

- A; 8 and 21
- B; 10 and 30
- C; 12 and 36
- D; 15 and 60
- E; 16 and 24
- F; 20 and 60

**Vocabulary**

- **term:** a number in a pattern.  
- **corresponding terms:** terms that have the same position in two related patterns.

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**English Language Learners: Differentiated Instruction**

**Levels 1–3**

**Speaking/Writing** Read *Connect It* problem 5. Display and model the meanings of the words *move along*, *grid lines*, *up*, *down*, *right*, and *left*. Remind students that directions are instructions that get us from one place, or point, to another. Have students point to the graph in the second *Model It*. Ask: What are the coordinates of the first point on the graph? What are the coordinates of the next point to its right? Have students form pairs. Ask students to take turns giving directions in writing to their partner to move between the points on the graph. Provide the following sentence frames to guide students: 

- **Move up _____ units and right _____ units.**

**Levels 2–4**

**Speaking/Writing** Ask students to choral read *Connect It* problem 5 with you. Have students point to the graph in the second *Model It*. Ask: What are the coordinates of the first point on the graph? What are the coordinates of the next point to its right? Move your finger to the right two units and up four units. What are the coordinates for this point? Have students form pairs. Ask students to take turns giving directions in writing to their partner to move between the points on the graph. Call on students to read their directions and have volunteers demonstrate how to find the coordinates on the graph.

**Levels 3–5**

**Speaking/Writing** Have students form pairs and read *Connect It* problem 5. Ask students to discuss the two patterns as shown in the table in the first *Model It*. Have students write sentences to explain the two patterns. Then ask pairs to take turns giving each other directions to move from one point on the graph to the next, moving from left to right. Extend the activity by having students choose a new point that is not yet marked on the graph. Have students write directions to explain how to find the point. Call on students to read their directions and have volunteers demonstrate how to find the coordinates on the graph.
The scouts are making model vehicles. They have a choice of making a model plane or a model boat.

- The materials for each plane cost $2.
- The materials for each boat cost $4.

Write two patterns to show the costs for making 0 to 4 of each type of vehicle. Graph points in the coordinate plane to show a relationship between corresponding terms of the patterns.

**Math Toolkit**
- grid paper
- base-ten blocks

**DISCUSS IT**

**Ask your partner:** Do you agree with me? Why or why not?
**Tell your partner:** I knew . . . so I . . .

**Sample A Sample B**

```
<table>
<thead>
<tr>
<th>Number of Vehicles</th>
<th>Plane Cost ($)</th>
<th>Boat Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
</tbody>
</table>
```

**Solution**

- A(0, 0), B(3, 2), C(6, 4)

**Listen for** To move from one point to the next, move right 3 units and up 2 units.

**Develop Language**

**Why** Reinforce the meaning of *each* as it is used to describe a singular unit.

**How** Ask students to circle the word *each* in *Try It*. Explain that the word *each* is used to describe a singular unit or thing. Point out that *each* does not refer to a specific unit, but rather any single unit. Invite students to suggest other words that could replace *each* in the problem (e.g., *the*, *a*, *any*, *every*, *one*). Have students tell how the substitutions change the meaning of the sentences.

**TRY IT**

**Make Sense of the Problem**

To support students in making sense of the problem, help them identify the costs of making a boat and the costs of making a plane.

**Ask** How many terms will be in each pattern?
Support Whole Class Discussion

**Compare and connect** the different representations and have students identify how they are related.

**Ask** How did you show the terms of the patterns? How did you label the coordinate plane and graph ordered pairs to show the relationship between corresponding costs of planes and boats?

**Listen for** Students should recognize that accurate responses include two patterns for the costs of making 0 to 4 of each vehicle and a coordinate plane graph that shows the relationship between the corresponding costs of the two vehicles. Responses may include ordered pairs with the cost of planes for one coordinate and cost of boats for the other. Students may also graph ordered pairs in the form (number of vehicles, cost of making that number of vehicles). In this case, students’ graphs will show two sets of points, with points for corresponding terms on the same vertical grid line (see student work Sample A in Try It). If students connect their points with lines, you may want to discuss that it is only the plotted points that have meaning in the problem context.

**MODEL IT**

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the cost of a number of planes and boats
- ordered pairs representing corresponding costs

**Ask** How do both the table and coordinate plane show ordered pairs?

**Listen for** The table shows ordered pairs \((x, y)\) in one column. The coordinate plane shows the ordered pairs as points on a graph.

For the **table model**, prompt students to consider how to use a table to form ordered pairs.

- Which columns of the table are used to form the ordered pairs in the last column?
- Each ordered pair is in the structure (cost of planes, cost of boats). Could you form ordered pairs in the structure (number of planes, cost of planes)?

For the **coordinate plane**, prompt students to consider how the ordered pairs in the table are shown in the graph.

- How do the titles on the horizontal and vertical axes relate to the table?
- How do you graph the ordered pair \((2, 4)\)?

**Deepen Understanding**

**Coordinate Plane Model**

**SMP 4** Model with mathematics.

When discussing the coordinate plane model, prompt students to consider how the scale and labels on each axis relate to the context of the problem.

**Ask** What scale is used on each axis? What does each grid line represent?

**Listen for** The scale for each axis is 1. Each grid line represents $1.

**Ask** How are the scale labels on the axes related to the costs in the problem?

**Listen for** The scale labels are multiples of 2 on each axis. They are the same as costs of the planes and boats, which are also multiples of 2.

**Ask** What if you use a scale of 2 on each axis? What would each grid line represent? How would using this scale change the graph?

**Listen for** Each grid line would represent $2. A graphed point would move closer to the origin. For example, the point \((2, 4)\) would be 1 grid line to the right and 2 grid lines up from \((0, 0)\) instead of 2 grid lines to the right and 4 grid lines up, as seen on the graph in Model It.
CONNECT IT

- Remind students that the one thing that is alike about the representations is that they both show the relationship between the cost of planes and the cost of boats.
- Explain that on this page, students will explore how the graph shows the relationship.

Monitor and Confirm

1. Check for understanding that:
   - the ordered pair (4, 8) shows the costs of making 2 planes and 2 boats
   - all ordered pairs show that the term for the cost of a number of boats is twice the term for the cost of the same number of planes
   - the ordered pairs are plotted correctly

Support Whole Class Discussion

4. Tell students that this problem will help prepare them to provide an explanation for problem 5.

Ask: What is the ordered pair that represents the costs of 5 planes and 5 boats? Why? Will the line go through it?

Listen for: The next ordered pair is (10, 20) because you add 2 to the x-coordinate and add 4 to the y-coordinate to reflect the costs of 1 additional plane and 1 additional boat. If you plot the point and extend the line, the line will go through (10, 20).

Have students note that while the line helps you see the pattern, not every point on the line has meaning in terms of the context of the problem.

Ask: Is the point (1, 2) on the line? Does this point have meaning in this problem situation?

Listen for: The point (1, 2) is on the line but since a plane costs $2 and a boat costs $4, there is no number of planes that cost $1 or number of boats that cost $2.

5. Look for the idea that moving from one point to the next point to the right reflects the rules for the patterns for the costs of planes and costs of boats.

6. Reflect

Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to use a graph to show relationships between corresponding terms of patterns.

1. Look at the table in the first Model It. What does the ordered pair (4, 8) represent? Possible answer: The x-coordinate shows the cost of building 2 planes, $4. The y-coordinate shows the cost of building 2 boats, $8.

2. How do the corresponding terms in the ordered pair (4, 8) compare? Is this relationship true of all corresponding terms in the table? Possible answer: The y-coordinate (the cost of 2 boats) is twice the x-coordinate (the cost of 2 planes). This is true for all the corresponding terms.

3. The first two ordered pairs in the table are graphed in the coordinate plane in the second Model It. Plot the other three ordered pairs from the table on the graph. See graph.

4. Connect the points on the graph in the second Model It. Explain what you see.

   Possible explanation: If you connect the points, they make a straight line.

5. Describe how to move along grid lines from one point to the next point to the right on your graph in the second Model It. How does your description relate to the rules for the patterns? Possible answer: Move 2 to the right and 4 up. The rule for the first pattern is add 2, so you would move right 2. The rule for the second pattern is add 4, so you would move up 4.

6. Reflect

   Look back at your Try It, strategies by classmates, and Model Its. Which models or strategies do you like best for how to use a graph to show relationships between corresponding terms of patterns? Explain.

   Some students may say they like to make a table to list the patterns and form ordered pairs of corresponding terms before graphing.

Visual Model

Change the graph by changing the form of the ordered pairs.

If . . . students think that there is only one correct graph,
Then . . . use this activity to compare two graphs for the same situation.

Materials: For display: empty table and coordinate plane similar to Model Its

- Tell students they are going to explore how the graph changes if they list the cost of boats as the first number in the ordered pair and the cost of planes as the second number in the ordered pair. Have students make a conjecture about how the graph will look. [For example: The points will still form a line.]
- Show the original table from the previous page. Make a new table with the cost of boats column and cost of planes column switched as well as the designations of x and y. List and graph the corresponding ordered pairs.
- Have a volunteer describe how the two graphs compare. [Possible answer: The line on the new graph is not as steep as the line on the original graph.]
- Ask students to explain whether the relationship between the cost of boats and the cost of planes changed. [Possible answer: The relationship did not change because the costs did not change.]
Apply It

For all problems, encourage students to make a table to help form ordered pairs if needed.

7. Consider the two patterns below. Start each pattern with 0.

Pattern A: add 1
Pattern B: add 3

Write five ordered pairs made up of corresponding terms from the two patterns. Plot the points in the coordinate plane to the right. Describe the relationship between the two patterns. Show your work.

Possible answer: (0, 0), (1, 3), (2, 6), (3, 9), (4, 12); Each term of Pattern B is 3 times the corresponding term of Pattern A.

8. In the school store, pencils are sold in packages of 6. Write five ordered pairs made up of corresponding terms of these two patterns, based on selling 0 to 4 packages of pencils.

Pattern A: number of packages sold
Pattern B: number of pencils sold

Plot the points in the coordinate plane. Describe the relationship between the coordinates of the ordered pairs. Show your work.

Possible answer: (0, 0), (1, 6), (2, 12), (3, 18), (4, 24); The y-coordinate is six times the x-coordinate.

9. Madeline and Javier graphed ordered pairs using the terms from the patterns they made. Madeline made the number pattern 0, 1, 3, 5, 7, 9. Javier made a number pattern with terms that were three times the corresponding terms in Madeline's pattern. Which list shows the ordered pairs Madeline and Javier graphed?

- A: (0, 0), (1, 3), (2, 6), (3, 9), (4, 12)
- B: (0, 0), (1, 3), (2, 6), (3, 9), (4, 12)
- C: (0, 0), (1, 3), (2, 6), (3, 9), (4, 12)
- D: (0, 0), (3, 1), (9, 3), (15, 5), (21, 7), (27, 9)

A; Multiply each term of Madeline's pattern by 3 to generate Javier's pattern: 0, 3, 9, 15, 21, 27. Identify the list of ordered pairs that uses the terms of Madeline's pattern as one coordinate and the corresponding terms of Javier's pattern as the other coordinate for all six ordered pairs.

Error Alert: If students choose B, C, or D, then have them make a table showing corresponding terms with Madeline's pattern as the x-coordinates and Javier's pattern as the y-coordinates. B incorrectly shows an “add 2” pattern for most of Javier's terms. C incorrectly shows a “four times” relationship between most corresponding terms of the two patterns. D mixes up terms in some ordered pairs. A table will help students organize patterns to minimize mistakes.
Luke compared a pattern with the rule “add 2” to a pattern with the rule “add 6.” He started at 0 and wrote the first three numbers of each pattern.

Add 2:
0, 2, 4

Add 6:
0, 6, 12

He wrote three ordered pairs.

\((0, 0)\), \((2, 6)\), \((4, 12)\)

Then he plotted the ordered pairs in the coordinate plane.

The first number in each ordered pair shows the location along the \(x\)-axis.

The second number in each ordered pair shows the location along the \(y\)-axis.
At a bake sale, cookies are sold in packages of 4 cookies each. Fruit bars are sold in packages of 2 fruit bars each.

3. Complete the table comparing the number of cookies and fruit bars sold for 0, 1, 2, and 3 packages.

<table>
<thead>
<tr>
<th>Number of Packages</th>
<th>Number of Cookies, x</th>
<th>Number of Fruit Bars, y</th>
<th>Ordered Pair (x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
<td>(4, 2)</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>4</td>
<td>(8, 4)</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>6</td>
<td>(12, 6)</td>
</tr>
</tbody>
</table>

4. Use the coordinate plane at the right to graph the ordered pairs from the table in problem 3. What directions would you give someone to get from one point to the next point to the right on the graph? Answers will vary. Possible answer: Move 4 to the right and 2 up.

5. How do your directions relate to the rules for the patterns? Answers will vary. Possible answer: The numbers in the rules are the same as the numbers in the directions. The rule for the cookies pattern is “add 4” and the rule for the fruit bars pattern is “add 2.” You move 4 to the right along the x-axis and 2 up along the y-axis to get to the next point.

6. Graph the next two points in the coordinate plane following these rules:
   - **Pattern A:** Start at 1. Multiply by 3. Subtract 1.
   - **Pattern B:** Start at 1. Multiply by 4. Subtract 1.

**English Language Learners:**

**Differentiated Instruction**

**Levels 1–3**

**Listening/Speaking** Read *Apply It* problem 8. Starting at the origin and moving from left to right, have students point to each of the four points on the graph and tell the x-coordinate for each point. Display the x-coordinates. Ask: What is the rule for Pattern A? What is the next term in the pattern? What will be the x-coordinate for the next point? Repeat the procedure for the y-coordinates and Pattern B.

Have students form pairs. Have pairs work together to extend the graph and plot a point to continue the patterns.

Call on students to tell or show how they know that the point they plotted continues the pattern.

**Levels 2–4**

**Speaking/Writing** Read *Apply It* problem 8 and have students choral read. Ask students to form pairs and discuss where the points on the graph are located. Have students write the patterns and rules for Pattern A and Pattern B. Encourage students to use a complete sentence for each. Then ask students to take turns giving their partner directions from the first point to the second point. Provide the following terms: move, x-axis, y-axis, and up. Call on students to say their directions.

Have pairs work together to extend the graph and plot a point to continue the patterns. Call on students to say their directions.

**Levels 3–5**

**Speaking/Writing** Have students form pairs and read *Apply It* problem 8. Ask students to discuss where the points on the graph are located. Have students write the patterns and rules for Pattern A and Pattern B. Then ask students to take turns giving their partner directions from the first to the second point. Provide the following terms: move, x-axis, y-axis, and up. Call on students to say their directions.

Have pairs work together to extend the graph and plot a point to continue the patterns. Have students write in complete sentences to explain how they determined the location of the new point.
LESSON 33

SESSION 4 Refine

Purpose In this session students solve word problems involving generating number patterns, identifying relationships between patterns, and writing ordered pairs for corresponding terms. They then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the Check for Understanding to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Why Confirm understanding of identifying relationships between number patterns.

How Have students find the relationship between the corresponding terms using any strategy they want.

Possible Solutions Each term in Pattern B is five times the corresponding term in Pattern A.

Describe the relationship between the corresponding terms of these two patterns.

Pattern A: 1, 3, 5, 7, 9
Pattern B: 5, 15, 25, 35, 45

Error Alert

If the error is . . .

Students may . . .

To support understanding . . .

<table>
<thead>
<tr>
<th>Error</th>
<th>Students May</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each term in Pattern B is 4 more than the corresponding term in Pattern A</td>
<td>Have subtracted the first term of Pattern A from the first term of Pattern B.</td>
<td>Have students read the first term of each pattern aloud and then tell how many times the first term of Pattern B is than the first term of Pattern A. Continue this process for the remaining terms, having students check that each term of Pattern B is always the same number of times the corresponding term of Pattern A.</td>
</tr>
<tr>
<td>Each term in Pattern B is 10 more than the corresponding term in Pattern A</td>
<td>Have identified the pattern in Pattern B.</td>
<td>Have students check their answer by adding 10 to each term in Pattern A and comparing the result to the corresponding term in Pattern B. Have them recognize that the results do not match the terms of Pattern B and then ask them to think about how to use multiplication to express a relationship between the corresponding terms of the two patterns.</td>
</tr>
</tbody>
</table>

EXAMPLE

Look at the following two number patterns.

Pattern A: 6, 5, 4, 3, 2, 1, 0
Pattern B: 24, 20, 16, 12, 8, 4, 0

What is the relationship between corresponding terms in the two patterns?

Look at how you could show your work using ordered pairs.

The first number is a term from Pattern A.
The second number is the corresponding term from Pattern B.

Ordered pairs: (6, 24), (5, 20), (4, 16), (3, 12), (2, 8), (1, 4), (0, 0)

Solution Each term in Pattern B is four times the corresponding term in Pattern A.

APPLY IT

One pattern starts at 0 and has the rule “add 8." Another pattern starts at 0 and has the rule “add 4." Write each pattern of numbers. How do the corresponding terms in the patterns compare? Show your work.

Add 8: 0, 8, 16, 24, 32, 40, ...
Add 4: 0, 4, 8, 12, 16, 20, ...

Solution Possible answer: The terms in the pattern with the rule “add 8" are twice the corresponding terms in the pattern with the rule “add 4.”

PAIR/SHARE How are these patterns different from other patterns in this lesson?

How can you generate the patterns?
EXAMPLE

Each term in Pattern B is four times the corresponding term in Pattern A; The ordered pairs shown is one way to solve the problem. Students could also solve the problem by making a table or drawing a diagram.

Look for The terms in most other patterns in this lesson increase in value. The terms in this pattern decrease in value.

APPLY IT

1. The terms in the pattern with the rule “add 8” are twice the corresponding terms in the pattern with the rule “add 4.” Students may also say that the terms in the pattern with the rule “add 4” are \(\frac{1}{2}\) times the corresponding terms in the pattern with the rule “add 8.” Students may make a table and write ordered pairs to compare corresponding terms.
   DOK 2
   Look for To generate the patterns, you can start at 0 and apply the rule “add 8” to generate one pattern and start at 0 and apply the rule “add 4” to generate the other pattern.

2. See completed table on the Student Worktext page. The \(y\) term is \(\frac{1}{4}\) times the corresponding \(x\) term, or the \(x\) term is 4 times the \(y\) term.
   DOK 2
   Look for The rule for the \(x\) column is “add 4,” and the rule for the \(y\) column is “add 1.”

3. D: Each term of the pattern of \(y\)-coordinates is 6 times the corresponding term of the pattern of \(x\)-coordinates.

Explain why the other two answer choices are not correct:

A is not correct because “10 more” is not true for (3, 18) or (4, 24).

B is not correct because the \(y\)-coordinate is not 2 times the \(x\)-coordinate in any of the 3 ordered pairs.

DOK 3

---

**Example**

Identify the pattern in each column of the table. Complete the \(x\)- and \(y\)-columns of the table. Use those columns to write ordered pairs in the last column. Describe the relationship between corresponding terms in the patterns. Show your work.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>Ordered Pair ((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>((4, 1))</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>((8, 2))</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>((12, 3))</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>((16, 4))</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>((20, 5))</td>
</tr>
<tr>
<td>24</td>
<td>6</td>
<td>((24, 6))</td>
</tr>
</tbody>
</table>

**Solution**

Possible answer: The \(y\) term is \(\frac{1}{4}\) times the corresponding \(x\) term.

3. The ordered pairs \((2, 12), (3, 18), (4, 24)\) are formed by corresponding terms in two patterns. How do the terms of the pattern of \(y\)-coordinates compare to the corresponding terms of the pattern of \(x\)-coordinates?

- A: 10 more
- B: 2 times as much
- C: \(\frac{1}{6}\) times as much
- D: 6 times as much

Mike chose D as the correct answer. How did he get that answer?

Mike compared the \(x\)-coordinates to the corresponding \(y\)-coordinates instead of comparing the \(y\)-coordinates to the corresponding \(x\)-coordinates.

---

Look for The terms in most other patterns in this lesson increase in value. The terms in this pattern decrease in value.

APPLY IT

1. The terms in the pattern with the rule “add 8” are twice the corresponding terms in the pattern with the rule “add 4.” Students may also say that the terms in the pattern with the rule “add 4” are \(\frac{1}{2}\) times the corresponding terms in the pattern with the rule “add 8.” Students may make a table and write ordered pairs to compare corresponding terms.

   DOK 2
   Look for To generate the patterns, you can start at 0 and apply the rule “add 8” to generate one pattern and start at 0 and apply the rule “add 4” to generate the other pattern.

2. See completed table on the Student Worktext page. The \(y\) term is \(\frac{1}{4}\) times the corresponding \(x\) term, or the \(x\) term is 4 times the \(y\) term.

   DOK 2
   Look for The rule for the \(x\) column is “add 4,” and the rule for the \(y\) column is “add 1.”

3. D: Each term of the pattern of \(y\)-coordinates is 6 times the corresponding term of the pattern of \(x\)-coordinates.

   Explain why the other two answer choices are not correct:
   A is not correct because “10 more” is not true for (3, 18) or (4, 24).
   B is not correct because the \(y\)-coordinate is not 2 times the \(x\)-coordinate in any of the 3 ordered pairs.

   DOK 3
4. Look at the patterns below. Choose True or False for each statement.

Pattern A: 3, 6, 9, 12, 15, 18, . . .
Pattern B: 18, 36, 54, 72, 90, 108, . . .

<table>
<thead>
<tr>
<th></th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>The rule for Pattern A is “multiply by 2.”</td>
<td>☑</td>
<td>☑</td>
</tr>
<tr>
<td>The rule for Pattern B is “add 18.”</td>
<td>☑</td>
<td>☑</td>
</tr>
<tr>
<td>Each term in Pattern A is 6 times the corresponding term in Pattern B.</td>
<td>☐</td>
<td>☑</td>
</tr>
<tr>
<td>Each term in Pattern B is 3 times the corresponding term in Pattern A.</td>
<td>☐</td>
<td>☑</td>
</tr>
</tbody>
</table>

5. Cindy and Dawn make these number patterns.


Write the first five terms of their patterns. Are there two corresponding terms that are the same? Explain. Show your work.

Solution
No; 16 is in both patterns, but it is the 3rd term of Cindy’s pattern and the 4th term of Dawn’s pattern. The 16s are not corresponding terms. See the list of five terms for each pattern on the Student Worktext page. Students may also make a table to show each pattern and write ordered pairs for corresponding terms.

Error Alert Students may not match up corresponding terms and say that the two 16s are corresponding terms. Encourage these students to organize the patterns in rows in a table and write corresponding terms as ordered pairs so that they can see that the 16 in Cindy’s pattern and the 16 in Dawn’s pattern are not in the same ordered pair. The 16 in Cindy’s pattern is the third term, but the 16 in Dawn’s pattern is the fourth term.

6. B; Students may label the change between one term and the next to identify the rule for each pattern.

DOK 1

Differentiated Instruction

RETEACH

Hands-On Activity

Use counters to compare two numerical patterns.

Students struggling with concepts of comparing corresponding terms of patterns Will benefit from additional work with concrete materials

Materials For each pair: 75 counters (45 of one color and 30 of a second color)

• Write the following patterns on the board:
  Pattern A: 1, 3, 5, 7, 9
  Pattern B: 2, 6, 10, 14, 18
• Tell students to begin with the color they have more of. Have them use one color counter to model the first term of each pattern, the other color to model the second term of each pattern, and so on, alternating colors.
• Ask students to describe the relationship between the two patterns. [Each term of Pattern B is twice the corresponding term in Pattern A.]
• Repeat the activity with other patterns.

EXTEND

Challenge Activity

Make numerical patterns.

Students who have achieved proficiency Will benefit from deepening understanding of patterns

Have students write two number patterns with at least 5 terms each so that all terms are whole numbers and each term of one pattern is \( \frac{3}{4} \) times the corresponding term of the other pattern.

• Have students check a partner’s pattern and then write a rule for each of their partner’s number patterns.
• Repeat the activity, changing \( \frac{3}{4} \) to \( \frac{5}{2} \).
Part A

See table on the Student Worktext page.

Part B

Students plot the ordered pairs from the table in the coordinate plane. See graph on the Student Worktext page. If correctly plotted, the points lie on the same line.

Part C

Each term in the “add 5” pattern is \(2\frac{1}{2}\) times the corresponding term in the “add 2” pattern.

DOK 2

Math Journal

Use the graph of corresponding terms of Pattern A and Pattern B. Write the first 5 terms of each pattern. Explain how you found the terms.

Pattern A: 0, 3, 6, 9, 12; Pattern B: 0, 2, 4, 6, 8; Possible explanation: I used the x-coordinates and y-coordinates of the points for the first 4 terms of the patterns. Then I could see that the rule for Pattern A is “add 3” and the rule for Pattern B is “add 2” and I used these rules to find the fifth terms.

Self Check

Go back to the Unit 5 Opener and see what you can check off.

Reinforce

Problems 4–8

Analyze number patterns.

All students will benefit from additional work with analyzing patterns by solving problems in a variety of formats.

• Have students work on their own or with a partner to solve the problems.
• Encourage students to show their work.

Personalize

Provide students with opportunities to work on their personalized instruction path with i-Ready Online Instruction to:

• fill prerequisite gaps
• build up grade-level skills

Close: Exit Ticket

Math Journal

Student responses should indicate understanding of how the terms of two patterns are given by a graph of corresponding terms in the coordinate plane. Students should be able to recognize from the axis titles which coordinates to use for Pattern A and which coordinates to use for Pattern B and then be able to extend the patterns to find the fifth terms.

Error Alert If students need additional support with finding the fifth terms in the patterns, then encourage them to describe how to move from one point to the next on the graph and to use this information to find the next term in each pattern.

Self Check Have students consider whether they feel they are ready to check off any new skills on the Unit 5 Opener page.
LESSON 33
Lesson Quiz

Tested Skills

Assesses 5.OA.B.3

Problems on this assessment form require students to be able to generate a numerical pattern given a rule, identify and describe relationships between corresponding terms of two patterns, and plot corresponding terms of two patterns as ordered pairs in the first quadrant of the coordinate plane. Students will also need to be familiar with using tables to identify patterns and list ordered pairs.

Alternately, teachers may assign the Digital Comprehension Check online to assess student understanding of this material.

Error Alert  Students may:
• misinterpret terms in the context of the problem.
• reverse the x- and y-coordinates of ordered pairs.
• describe a relationship that is true for some, but not all, pairs of corresponding terms of patterns.

Solutions

1  C; Students could solve the problem by adding 16 to each consecutive term in Pattern B.
   A is not correct because the rule “multiply by 2” does not apply to each term in Pattern A.
   B is not correct because the rule “each term in Pattern A is 12 less than the corresponding term in Pattern B” applies only to the first term in Pattern A and Pattern B.
   D is not correct because the terms in Pattern B are 4 times the corresponding terms in Pattern A, not 2 times the corresponding terms.
   1 point  5.OA.B.3, DOK 1

2 Mason makes number patterns. Pattern A starts at 0 and has the rule “add 3”.
   Pattern B starts at 0 and has the rule “add 9”. Write the first five terms of each pattern. How are the corresponding terms related? Show your work. (2 points)
   Possible student work:
   Pattern A: 0, 3, 6, 9, 12
   Pattern B: 0, 9, 18, 27, 36
   Solution  The terms in Pattern B are three times the corresponding terms in Pattern A.

3 An ice cream stand charges $2 for a kiddie cone and $6 for a large cone. Clara makes two patterns to compare the costs. She writes ordered pairs in the form (kiddie cone cost, large cone cost) for the corresponding numbers of cones.
   Which ordered pairs could be on Clara’s list? Choose all the correct answers. (2 points)
   A (6, 12)  B (8, 24)  C (10, 14)  D (12, 36)  E (16, 64)

Graphing Points Scoring Rubric

<table>
<thead>
<tr>
<th>Points</th>
<th>Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Response contains the following: • Correct computations, solutions, and/or calculations. (1 point) • Well-organized, clear, and concise work that demonstrates thorough understanding of math concepts and/or procedures. (1 point)</td>
</tr>
<tr>
<td>1</td>
<td>Response contains the following: • Mostly correct solution(s). • Shows partial or good understanding of math concepts and/or procedures.</td>
</tr>
<tr>
<td>0</td>
<td>Response contains the following: • Incorrect solution(s). • No attempt at finding a solution. • No effort to demonstrate an understanding of mathematical concepts and/or procedures.</td>
</tr>
</tbody>
</table>

Multiple Select Scoring Rubric

<table>
<thead>
<tr>
<th>Points</th>
<th>Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>All answers are correct</td>
</tr>
<tr>
<td>1</td>
<td>1 incorrect answer</td>
</tr>
<tr>
<td>0</td>
<td>2 or more incorrect answers</td>
</tr>
</tbody>
</table>
**Lesson 33 Quiz continued**

4 At the library book sale, paperback books cost $2 and hardcover books cost $3.
Dominic makes this table to compare the costs of the two types of books.
Use this table to solve the problems in Part A and Part B.

<table>
<thead>
<tr>
<th>Number of Books</th>
<th>Cost of Paperback Books (in dollars), x</th>
<th>Cost of Hardcover Books (in dollars), y</th>
<th>Ordered Pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>(2, 3)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
<td>(4, 6)</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>9</td>
<td>(6, 9)</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>12</td>
<td>(8, 12)</td>
</tr>
</tbody>
</table>

Part A
Graph the ordered pairs from the table. (2 points)

Part B
Describe the relationship between the corresponding terms of the pattern for the cost of paperback books and the pattern for the cost of hardcover books. (2 points)

Possible answer: Each term in the pattern for the cost of hardcover books is \( \frac{3}{2} \) times the corresponding term in the pattern for the cost of paperback books.

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**Differentiated Instruction**

**RETEACH**

**Tools for Instruction**

Students who require additional support for prerequisite or on-level skills
Will benefit from activities that provide targeted skills instruction

**REINFORCE**

**Math Center Activities**

Students who require additional practice to reinforce concepts and skills and deepen understanding
Will benefit from small group collaborative games and activities (available in three versions—on-level, below-level, and above-level)

**EXTEND**

**Enrichment Activities**

Students who have achieved proficiency with concepts and skills and are ready for additional challenges
Will benefit from group collaborative games and activities that extend understanding