Lesson Objectives

Content Objectives

- Classify two-dimensional figures in a Venn diagram or tree diagram based on properties of the figures.
- Draw and use Venn diagrams and tree diagrams to show the relationships among categories of two-dimensional figures.

Language Objectives

- Discuss the key term attribute and its meaning with a partner.
- Explain how Venn diagrams and tree diagrams show the relationships between categories.
- Describe relationships among two-dimensional figures shown in Venn diagrams and tree diagrams.

Prerequisite Skills

- Recognize parallel and perpendicular lines.
- Recognize right, acute, and obtuse angles.
- Recognize that triangles can be classified based on the lengths of their sides (isosceles, equilateral, scalene).
- Sort two-dimensional figures based on the kinds of sides they have and on the kinds of angles they have.
- Understand how a Venn diagram and a tree diagram show category-subcategory relationships.

Learning Progression

In Grade 4 students classified two-dimensional figures based on their attributes, such as having parallel or perpendicular sides and having right, acute, or obtuse angles. In the previous Grade 5 lesson students described how attributes belonging to a larger category belong to subcategories of that category and they used simple Venn diagrams and tree diagrams to show category/subcategory relationships.

In this lesson students use their understanding of shape properties, categories, and subcategories to classify shapes into Venn diagrams and tree diagrams, including Venn diagrams that have regions that partially overlap.

In Grade 6 students will work with figures in the coordinate plane and will find the area of two-dimensional figures.

Lesson Vocabulary

- **trapezoid** (exclusive) a quadrilateral with exactly one pair of parallel sides.
- **trapezoid** (inclusive) a quadrilateral with at least one pair of parallel sides.
- **attribute** any characteristic of an object or shape, such as number of sides or angles, lengths of sides, or angle measures.
- **category** a collection of objects grouped together based on attributes they have in common.
- **hierarchy** a ranking of categories based on attributes.
- **polygon** a two-dimensional closed figure made with three or more straight line segments that do not cross over each other.
- **subcategory** a category within a larger category. It shares all the same attributes as the larger category. For example, parallelograms are a subcategory of quadrilaterals.
- **tree diagram** a hierarchy diagram that connects categories and subcategories with lines to show how they are related.
- **Venn diagram** a diagram that uses overlapping ovals (or other shapes) to show how sets of numbers or objects are related.

CCSS Focus

**Domain**

Geometry

**Cluster**

B. Classify two-dimensional figures into categories based on their properties.

**Standard**

5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties.

**Additional Standard**

5.G.B.3 (See Standards Correlations at the end of the book for full text.)

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the Try-Discuss-Connect routine.*

In addition, this lesson particularly emphasizes the following SMPs:

- **5** Use appropriate tools strategically.
- **7** Look for and make use of structure.

*See page 305m to see how every lesson includes these SMPs.
**Lesson Pacing Guide**

### Whole Class Instruction

<table>
<thead>
<tr>
<th>SESSION 1</th>
<th>Explore</th>
<th>45–60 min</th>
<th><strong>Interactive Tutorial</strong> (Optional)</th>
<th>Additional Practice</th>
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<tr>
<td></td>
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<td></td>
<td><strong>Prerequisite Review:</strong> Identify Two-Dimensional Figures</td>
<td>Lesson pages 589–590</td>
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<td><strong>Classifying Two-Dimensional Figures</strong></td>
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<td>• Close: Exit Ticket 5 min</td>
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<tr>
<th>SESSION 2</th>
<th>Develop</th>
<th>45–60 min</th>
<th><strong>Classifying Two-Dimensional Figures</strong></th>
<th>Additional Practice</th>
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<tr>
<th>SESSION 3</th>
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<th><strong>Classifying Two-Dimensional Figures with Tree Diagrams</strong></th>
<th>Additional Practice</th>
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<tr>
<th>SESSION 4</th>
<th>Refine</th>
<th>45–60 min</th>
<th><strong>Classifying Two-Dimensional Figures</strong></th>
<th>Lesson Quiz or Digital Comprehension Check</th>
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<td>• Example &amp; Problems 1–3 15 min</td>
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<td>• Practice &amp; Small Group Differentiation 20 min</td>
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<td>• Close: Exit Ticket 5 min</td>
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### Small Group Differentiation

**PREPARE**

**Ready Prerequisite Lesson**

- Grade 4
  - Lesson 33 Classify Two-Dimensional Figures

**RETEACH**

**Tools for Instruction**

- Grade 4
  - Lesson 33 Attributes of Shapes

- Grade 5
  - Lesson 29 Classify Plane Figures

**REINFORCE**

**Math Center Activities**

- Grade 5
  - Lesson 29 Organize Polygons on a Venn Diagram

  - Lesson 29 Organize Triangles on a Venn Diagram

**EXTEND**

**Enrichment Activity**

- Grade 5
  - Lesson 29 Sorting Shapes

### Independent Learning

**PERSONALIZE**

**i-Ready Lesson**

- Grade 5
  - Classify Two-Dimensional Figures

### Lesson Materials

**Lesson** *(Required)*: none

**Activities**:

- **Per pair**: geoboard, rubber bands
- **Per group**: 1 set of index cards, each showing a parallelogram, square, or rectangle (prepared in various sizes, colors, orientations), set of 5 sticky notes with letters on them (blue X, red X, green X, blue Y, blue Z), set of 10 sticky notes with vegetable categories written on them (peppers, carrots, vegetables, potatoes, red pepper, hot peppers, seeded vegetables, green peppers, root vegetables, and sweet potatoes), large sheet of paper, markers, blank sticky notes
- **For display**: masking tape, Venn diagram with ovals for Triangles, Acute Triangles, and Isosceles Triangles (see Session 3 Hands-On Activity for details)

**Math Toolkit**: geoboard, rubber bands, tracing paper, grid paper, rulers

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*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.*
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

**Connect to Family**

Use the Family Letter—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

**Goal**

The goal of the Family Letter is to provide opportunities for family members to support students as they learn more about how two-dimensional figures are related and classified.

**Activity**

In the Classifying Two-Dimensional Figures activity, students and family members use Venn diagrams to show how figures are related and describe the relationships using the figures’ attributes.

**Math Talk at Home**

Encourage students to continue to talk with their family members about attributes that can be used to describe two-dimensional figures and discuss the relationships between them. Remind students that two-dimensional figures can be classified into categories and subcategories based on their attributes.

**Conversation Starters**

Below are additional conversation starters children can write in their Family Letter or math journal to engage family members:

- What are some ways you classify objects at work or at home?
- What are some objects you can classify by size? by shape?
### Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

**Session 1** Use with *Additional Practice*, problem 3.

- Discuss with students how triangles can be categorized by their sides and angles. Encourage students to think about objects shaped like triangles. Invite volunteers to provide examples. Some examples of objects might include building blocks, tortilla chips, sails from a sailboat, and folded napkins. Have students support their answers by describing the attributes of each object.

### Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

<table>
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<tr>
<th>Levels 1–3</th>
<th>Levels 2–4</th>
<th>Levels 3–5</th>
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</table>
| **Speaking/Reading** Read the *Try It* problem aloud. Organize students into triads. Have each group member model one of the shapes on a geoboard. Ask them to use gestures, words, and pictures to discuss the attributes of each shape, then work together to create a Venn diagram. Provide the following sentence starters to guide small group conversation.  
  - *Squares are a type of* ___________.  
  - *Rectangles are a type of* ___________.  
  - *Rectangles and squares both are* ___________. | **Speaking/Listening** Read the *Try It* problem with students. Have them form pairs to restate the problem in their own words and determine the category words that can be used to label a Venn diagram to arrange the quadrilaterals. Provide the following sentence starters for partners to complete:  
  - *Squares are a subcategory of* ___________.  
  - *Rectangles are a subcategory of* ___________.  
  - *Rectangles and squares are both subcategories of* ___________.  
  After partners complete their Venn diagram, ask them to explain how the diagram shows the categories ranked from most general to most specific. | **Speaking/Writing** Have students read and solve *Try It* with a partner. Have partners discuss the relationships between the categories, including describing one category as a subcategory of another. Encourage them to write their explanations using complete sentences and precise mathematical language. Ask students to share and compare their explanations with another pair. |
| **English Language Learners: Differentiated Instruction** Prepare for Session 1 Use with *Try It.* |
In this session students draw on their understandings of categories and subcategories of two-dimensional shapes to arrange given quadrilaterals into a Venn diagram. They share their drawings of Venn diagrams with one another to explore how the shapes share attributes. They will look ahead to think about how Venn diagrams can use partially overlapping regions to show that two categories of shapes share properties even though neither category is a subcategory of the other.

**Start**

**Connect to Prior Knowledge**

**Why** Review sorting shapes based on their properties to prepare for classifying shapes using Venn diagrams.

**How** Have students record their groups on their papers using shape numbers. Encourage early finishers to sort in more than one way. Have students share and compare their sorts.

**Possible Solutions**

Divide the shapes into two groups. Give each group a title to explain your grouping.

Group 1: Shapes with a right angle (1, 5, 6);
Group 2: Shapes without a right angle (2, 3, 4);
Group 1: Quadrilaterals (2, 3, 5, 6); Group 2: Not Quadrilaterals (1, 4)

**TRY IT**

**Make Sense of the Problem**

To support students in making sense of the problem, help them understand each shape’s name as well as the task of drawing a Venn diagram.

**DISCUSS IT**

**Support Partner Discussion**

To reinforce the language of hierarchy, encourage students to use attributes, properties, category, and subcategory as they talk to each other. Look for, and prompt as necessary for, understanding of:

- the shapes are rectangle, parallelogram, and square
- Venn diagrams show a subcategory with an oval entirely contained within the oval for the broader category

**Common Misconception**

Look for students who are not comfortable categorizing the shapes from general to specific. As students present solutions, have them specify the properties of each shape.

**Select and Sequence Student Solutions**

One possible order for whole class discussion:

- concrete models or drawings to support reasoning about properties
- lists or tables of properties along with a Venn diagram
- Venn diagrams with categories from parallelograms to rectangles to squares
- Venn diagrams that include the broader category of quadrilaterals

**Support Whole Class Discussion**

Prompt students to explain the relationships shown by their diagrams.

**Ask** How do [student name]’s and [student name]’s Venn diagrams show how to order the shapes from the most general category to the most specific subcategory?

**Listen for** The most general category is quadrilaterals because all 3 shapes have 4 sides and 4 angles. The first subcategory is parallelograms, which also have 2 pairs of parallel sides of equal length. Next are rectangles, which also have 4 right angles. The most specific subcategory is squares, which also have 4 sides of equal length.
**CONNECT IT**

1 **LOOK BACK**

Look for understanding that the shapes are organized in a hierarchy starting with the broader category of quadrilaterals (or parallelograms if students don’t include quadrilaterals) to the most specific subcategory of squares.

- **Visual Model**
  
  Make a class Venn diagram.

  *If . . . some students are unsure about ordering figures by properties in a Venn diagram,*

  *Then . . . use this activity to have them act out sorting and classifying shapes.*

  **Materials** For each group: 1 set of index cards, each showing pictures of a parallelogram, square, or rectangle (prepared in various sizes, colors, orientations); For display: masking tape

  - In an open floor space, use tape to outline a nested Venn diagram with regions for parallelograms, rectangles, and squares.
  - Give each student an index card with a shape.
  - Have the group stand in the outside of the oval for parallelograms. Ask: *Look at your shape. Why do you belong in the Parallelograms region? What properties do you all share?* [4 sides, 2 pairs of parallel sides, 2 pairs of sides with equal length]
  - Tell students to move to the Rectangles region if they think their shape has the properties of a rectangle. Ask: *What property do you have that the other parallelograms do not have?* [4 right angles] Do you still share the other properties of a parallelogram? [yes]
  - Tell students to move to the Squares region if they think their shape has the properties of a square. Ask: *What property do you have that other rectangles do not have?* [4 equal sides]
  - Have students pick new cards and repeat.

2 **LOOK AHEAD**

Point out that Venn diagrams can also show relationships between shapes that have some properties in common but neither has all the properties of the other. Students should be able to name properties the shapes share and why the equilateral triangle belongs in the region of overlap.

3 **REFLECT**

Look for understanding that the ovals for categories overlap without one being fully contained in the other when each shape shares some but not all properties of the other shape and that the oval for a subcategory is inside the oval for another category when the shape has all properties of the broader category.

**Common Misconception** If students do not use precision in their explanations and just say categories that overlap share properties, *then* have them describe how they share properties in more detail. Ask if they share all properties, some properties, or no properties. Look for students to describe that categories that overlap share one or more properties but not all properties.

**Real-World Connection**

Encourage students to think about everyday places or situations in which people can use the concept of ordering from most general to most specific. For example, think about answering the question “Where do you live?” You could answer based on your continent, country, state, city, town, neighborhood, or street name. All answers would be accurate, but the name of your continent would be the most general answer and the name of your street would be the most specific answer.
1. Think about what you know about Venn diagrams. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. Possible answers:

<table>
<thead>
<tr>
<th>What Is It?</th>
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<tr>
<td>a drawing that uses overlapping ovals to show how categories of shapes relate</td>
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</table>

<table>
<thead>
<tr>
<th>What I Know About It</th>
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<tbody>
<tr>
<td>When the ovals in a Venn diagram overlap, it means that a shape that belongs in the area of overlap can be called by the category labels of both ovals.</td>
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</table>

2. Look at the Venn diagram. Why is the square inside of all the ovals? Possible answer: Squares have all of the properties of both rectangles and parallelograms, so they are also rectangles and parallelograms.
Assign problem 3 to provide another look at using Venn diagrams to classify two-dimensional figures.

This problem is similar to the problem about arranging quadrilaterals in a Venn diagram. In both problems, students are asked to use a Venn diagram to show relationships between the categories of shapes. The question asks students to arrange the given triangles in a Venn diagram for which the structure is provided and to label the ovals with titles that describe the categories.

Students may want to use geoboards, tracing paper, grid paper, or rulers.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- What is this problem about?
- What is the question I am trying to answer?
- What information is important?

**Solution:** See the completed Venn diagram on the student page. Students may also switch the positions of the left and right triangles and reverse the labels for the ovals.

**Medium**

Have students check their answer in another way, such as describing the reasoning they used to arrange the figures in the diagram.

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**English Language Learners:**

**Differentiated Instruction**

**Levels 1–3**

**Speaking/Writing** Read *Connect It* problem 6. Review the *Model Its* with students. Provide the following terms and have the students repeat them after you: *table, circles, inner circles.*

Help students use the words to tell which model or strategy they like the best. Provide the following sentence starter:

- *I like using _____.*

Have students copy the complete sentence and read it to a partner.

**Levels 2–4**

**Speaking/Writing** Read *Connect It* problem 6. Pair students and have them discuss the similarities and differences between the models and strategies in the *Model Its*. Ask students to explain to their partners which model or strategy they like best.

Provide a sentence frame to guide students during their conversation and ask them to write their ideas:

- *I like the _____ because _____.*

After students complete their sentences, have them take turns reading them to their partners.

**Levels 3–5**

**Speaking/Writing** Have pairs read *Connect It* problem 6. Ask partners to make a list of similarities and differences between the models and strategies in the *Model Its* as well as strategies they used in *Try It*.

Have students use the list to determine which model or strategy they prefer to use. Ask students to explain their selection in writing.
In this session students solve a problem that requires sorting quadrilaterals into a Venn diagram. The purpose of this problem is to have students develop strategies for sorting shapes in a more complex Venn diagram that includes partially overlapping ovals.

Classify Two-Dimensional Figures

Read and try to solve the problem below.

Sort these shapes into the Venn diagram below to show the relationships among parallelograms, squares, rectangles, rhombuses, and quadrilaterals.

Use at least one shape in each region of the diagram. Classify the shapes by labeling each region with the category name.

Possible Solutions
1: triangle, equilateral triangle;
2: rectangle, square;
3: polygon, quadrilateral;
In each case, the second category is a subcategory of the first.

Possible student work:
Sample A
Sample B
Note: Students may choose to draw each shape in the Venn diagram.

Develop Language
Why Reinforce understanding of the word hierarchy.
How Remind students that a hierarchy is a way of classifying objects by ranking them in a connected order based on certain properties or attributes. Provide examples of hierarchies in everyday life, such as a family tree or a simple organizational chart of your school. Have students make connections and explain the relationships represented by the hierarchy.

TRY IT
Make Sense of the Problem
To support students in making sense of the problem, point out that the blank Venn diagram in the Try It space will be used in the problem. Be sure students understand that they are both identifying the category name for each region and placing specific shapes into those regions.
Ask How many categories does the Venn diagram have regions for?

DISCUSS IT
Support Partner Discussion
Encourage students to use the term property as they discuss their solutions. Support as needed with questions such as:
• How did you get started?
• Did you move shapes from one oval to another as you solved the problem? Why?

Common Misconception Look for students who put the rhombus in the center of the middle overlapping ovals and the square and rectangles on the outside of these ovals. Ask them to list the properties of each shape.

Select and Sequence Student Solutions
One possible order for whole class discussion:
• concrete models or drawings to support reasoning about properties
• lists or tables of properties
• shapes drawn in the Venn diagram
• letters used to represent shapes in the Venn diagram
Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask  How does your model show the relationships among the shapes? Is there more than one correct way to place the shapes in the Venn diagram?

Listen for  Students should recognize Venn diagrams that show correct category/subcategory relationships of the shapes. Representations of properties may be in lists or tables, and Venn diagrams may include letters or shapes drawn in the regions of the diagram as well as the category labels for each oval and the region of overlap. Rhombuses and rectangles may be on either side of the region of overlap at the center of the diagram.

MODEL ITS

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

• each shape in the problem
• relationships between shapes

Ask  How are the shapes shown in the problem represented in each Model It?

Listen for  In the first Model It, the shape names are shown in a table with properties of each shape identified by Xs. The second Model It shows the shapes sorted into a Venn diagram.

For the table model, prompt students to consider a table as an aid to identify properties.

• How does the table show the properties a shape has? the properties a shape does not have?
• How might using this type of table help you make a Venn diagram?

For the Venn diagram model, prompt students to relate the Venn diagram to the table.

• How does the placement of a shape in the Venn diagram relate to the number of Xs shown for that shape in the table?
• Rectangles and Rhombuses each have four Xs: three for the same properties and one for different properties. What does this mean about the ovals for Rectangles and Rhombuses in the Venn diagram?

Deepen Understanding

Table Model

SMP 7  Use structure.

When discussing the table model, prompt students to consider how the number of Xs in each column can help them order the shapes from most general to most specific.

Ask  How do the number of Xs show which shape is the most general category?

Listen for  Quadrilaterals have one X for the attribute 4 sides and no Xs for other more specific properties, so quadrilaterals are the most general category.

Ask  How do the number of Xs show which shape is the most specific subcategory?

Listen for  Squares are the only shape with Xs for every property. So, squares are the most specific subcategory.

Ask  If you rearranged the columns from most general to most specific, what pattern would you see in the number of Xs?

Listen for  The number of Xs would increase, or stay the same, from left to right.
**CONNECT IT**

- Remind students that one thing that is alike about the representations is that they show how the shape categories relate to each other.
- Explain that on this page, students will use their knowledge of properties of shapes to understand the category relationships shown by overlapping regions in a Venn diagram.

**Monitor and Confirm**

1 – 3 Check for understanding that:

- quadrilaterals are the broadest category because they share only one property with other shapes
- the relationships between ovals in the Venn diagram show how the categories of shapes are related
- shapes within an oval that is completely inside another oval have all the properties of the shapes represented by the larger oval

**Support Whole Class Discussion**

4 Tell students that this problem will prepare them to provide the explanation required in problem 5.

**Ask** Look at the overlapping ovals for Rhombuses and Rectangles again. What does it mean if a shape goes in the region of overlap?

**Listen for** The shape is both a rectangle and a rhombus, having the properties of both rectangles and rhombuses.

5 Look for the idea that a square has the most properties, sharing all of the properties of the shapes represented by the larger ovals.

**REFLECT** Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

**Hands-On Activity**

**Sort objects that are in overlapping categories.**

If . . . students are unsure about Venn diagrams with overlapping ovals,

Then . . . use this activity to help them understand the region of overlap.

**Materials**

For each group: set of 5 sticky notes with letters on them (blue X, red X, green X, blue Y, blue Z); large sheet of paper; markers; blank sticky notes

- Have each group draw two overlapping ovals on their paper. Distribute sets of prepared sticky notes to the groups.
- Tell groups to sort the sticky notes into their two ovals based on properties. Have then write a category name for each oval, based on their sort.
- Ask: What are the categories for your two ovals? Why? [Xs and Blue Letters; all the sticky notes show an X and/or a blue letter]
- Ask: Which sticky notes are in the region of overlap? Why? [blue Xs; blue Xs have both properties, X and blue letter]
- If time allows, give groups blank sticky notes and have them make a set of letters or shapes that can be sorted into two overlapping categories.
**APPLY IT**

For all problems, encourage students to think about and possibly list the properties of the shapes involved as they work with the Venn diagrams. Problem 7 is an opportunity to draw students’ attention to the inclusive and exclusive definitions of trapezoid, explaining that either definition is acceptable but only one definition can be used in any given situation. In any discussion or problem situation dealing with trapezoids, it is important to know which definition is in effect.

**7 Left:** at least one pair; If a trapezoid is defined as having at least one pair of parallel sides, parallelograms and rectangles are both subcategories of trapezoids because they also have at least one pair of parallel sides. Note: this is the inclusive definition of trapezoid.

**Right:** exactly one pair; If a trapezoid is defined as having exactly one pair of parallel sides, trapezoids do not overlap at all with parallelograms and rectangles, because parallelograms and rectangles have more than one pair of parallel sides. Note: this is the exclusive definition of trapezoid.

**8** See the completed Venn diagram on the Student Worktext page; Students may show labels for “Shapes with Parallel Sides” and “Shapes with Perpendicular Sides” on either oval.

**Close: Exit Ticket**

**9** The definition that a trapezoid has at least one pair of parallel sides; See possible explanation on the Student Worktext page.

Students’ solution should indicate understanding of:
- the properties of a parallelogram
- the properties of a trapezoid
- the difference between the inclusive and exclusive definitions of trapezoid

**Error Alert** If students’ solution is exactly one pair of parallel sides, then review the meaning of parallel. Have students look at the sides of shape A again after your review and have them trace it as necessary to help determine the number of pairs of parallel sides it has.
Practice Classifying Two-Dimensional Figures

Study the Example showing how to use Venn diagrams to classify two-dimensional figures. Then solve problems 1–5.

**EXAMPLE**

You can use a Venn diagram to show the relationships between acute, right, obtuse, isosceles, and equilateral triangles.

You can plan your Venn diagram using a table describing properties of the triangles.

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute</td>
<td>all acute angles</td>
</tr>
<tr>
<td>Right</td>
<td>2 acute angles and 1 right angle</td>
</tr>
<tr>
<td>Obtuse</td>
<td>2 acute angles and 1 obtuse angle</td>
</tr>
<tr>
<td>Isosceles</td>
<td>at least 2 sides of equal length</td>
</tr>
<tr>
<td>Equilateral</td>
<td>all sides equal length</td>
</tr>
</tbody>
</table>

1. Look at the Venn diagram above. Can a right triangle ever be an equilateral triangle? Explain.

   **No:** Possible explanation: Right triangles do not overlap at all with equilateral triangles. Right triangles always have a right angle and equilateral triangles never have a right angle.

**Fluency & Skills Practice**

**Assign Classifying Two-Dimensional Figures**

In this activity students use Venn diagrams to classify triangles and quadrilaterals. This practice sharpens students’ abilities not only to see obvious differences between shapes but also to identify subtler similarities. For example, students may note that squares and rhombuses both have 4 sides of equal length. They may also see that right triangles, rectangles, and some trapezoids have at least 1 right angle.
See two possible statements on the student page; Students need only provide one statement. Students may also say that not all acute triangles are isosceles triangles. All explanations reflect that the ovals for acute triangles and isosceles triangles overlap, but neither is fully contained within the other.

Medium

See one possible statement on the student page; Students may also say that not all acute triangles are equilateral triangles. All explanations reflect that the oval for equilateral triangles is completely inside the acute triangles oval.

Medium

See the Venn diagram on the student page. The oval for scalene triangles should not touch the ovals for isosceles and equilateral triangles, because those types of triangles have at least 2 sides of equal length.

Medium

A (True);
D (False);
F (False);
G (True)

Challenge

1. Look at the Venn diagram on the previous page. Write a statement about the relationship between acute triangles and isosceles triangles.

Possible answer: An isosceles triangle can be an acute triangle if all of the angles in the isosceles triangle are acute. Not all isosceles triangles have angles that are all acute.

2. Look at the Venn diagram on the previous page. Write a statement about the relationship between acute triangles and equilateral triangles.

Possible answer: An equilateral triangle is always an acute triangle because all of the angles in an equilateral triangle are acute.

3. Draw a Venn diagram in the space below to show the relationships among the categories of isosceles, scalene, and equilateral triangles within the broader category, Triangles.

4. Determine whether each statement is True or False. Draw a picture to help if needed.

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>A scalene triangle is never isosceles.</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td>A right triangle is sometimes equilateral.</td>
<td>✗</td>
<td>✓</td>
</tr>
<tr>
<td>A right triangle is never isosceles.</td>
<td>✗</td>
<td>✓</td>
</tr>
<tr>
<td>A scalene triangle can be a right, obtuse, or acute triangle.</td>
<td>✗</td>
<td>✓</td>
</tr>
</tbody>
</table>

ELL

English Language Learners: Differentiated Instruction

Prepare for Session 3

Use with Try It.

Levels 1–3

Speaking/Writing Read the Try It problem aloud to students. Display a 2 × 8 table with the headings Shapes and Properties. Ask students to identify the shapes to be classified. Record the shapes on the table. As a group, fill in the properties for trapezoid. Discuss the definition provided in the problem.

Organize students into small groups and have them copy and complete the table. Give each group seven index cards. Ask them to label each index card with the name and picture of a shape to be classified. Have group members lay the cards out on a table, think aloud about their properties, and manipulate the cards to arrange their tree diagram.

Levels 2–4

Speaking/Writing Read the Try It problem with students. Have students form pairs to discuss and solve the problem. Ask them to create a table of the shapes and describe their properties.

Have partners work use index cards with the names of each shape on them to organize their tree diagram.

Ask each pair to determine how many categories they used to classify the shapes. Call on partners to share their categories. Then allow time for students to revise their tree diagrams based on the comments of others.

Levels 3–5

Speaking/Writing Have students read the Try It problem independently and plan their tree diagram using a table to describe the properties of the shapes.

Organize students into pairs to share and compare their tables. Ask students to critique their partner’s work, ask questions, and point out possible mistakes. Then have partners work together to create the tree diagram.
Purpose
In this session students solve a problem that requires classifying shapes using a tree diagram. The purpose of this problem is to have students develop an understanding of the relationships among figures that may be classified into more than one category, and to display categories in a tree diagram to show hierarchy based on the properties of the shapes.

Start

Connect to Prior Knowledge
Why Prepare students for classifying shapes into a tree diagram by recognizing a hierarchy of categories.
How Have students identify attributes that shapes share and attributes that are unique.

Possible Solutions
Name two attributes these shapes share and two attributes these shapes do not share.

 Develop Language
Why Analyze the antonyms inclusive and exclusive to reinforce understanding of the two definitions of trapezoid.
How Discuss the everyday meanings of the words include and exclude, then display the two definitions of trapezoid. Guide students to identify which shapes are excluded by the definition used in the Try It problem (parallelograms, rhombuses, rectangles and squares).

TRY IT
 Make Sense of the Problem
To support students in making sense of the problem, have them restate the two definitions of trapezoid that they have seen, and clarify which definition of trapezoid is used in this problem.
Ask What are the two different definitions of trapezoids and which definition is used in this problem?

DISCUSS IT
Support Partner Discussion
Encourage students to use the terms property and attribute as they discuss their solutions.
Support as needed with questions such as:
• How did you get started?
• How did you organize the information in the problem?

Common Misconception Look for students who include trapezoid in the path between quadrilateral and parallelogram. Ask them the definition of trapezoid and listen for the words at least one pair of parallel sides or exactly one pair of parallel sides.

Select and Sequence Student Solutions
One possible order for whole class discussion:
• concrete models or drawings to support reasoning about attributes
• list or table of attributes
• hierarchy of categories shown in a tree diagram
• classification of shapes A and B
Support Whole Class Discussion

**Compare and connect** the different representations and have students identify how they are related.

**Ask** How does your tree diagram show the relationship between categories of shapes? How can you find all classifications for each shape using your diagram?

**Listen for** Students should recognize how tree diagrams can be used to show the hierarchy of shapes. In both models, the categories move from most general to most specific. For example, Figure A is a rhombus. Student may follow the tree diagram either left or up from rhombus to find less specific categories to which all rhombuses also belong. Listen for clarification that as you move down or to the right on the tree diagram, the classifications become more specific, and as you move up or left the classifications are more general.

**MODEL ITS**

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- each attribute of the listed shapes
- relationships between categories of shapes

**Ask** How are the shapes in the problem represented in each Model It?

**Listen for** In the first Model It, the shape categories are shown in a Venn diagram. In the second Model It categories of shapes are shown in a tree diagram.

**For the Venn diagram model**, prompt students to consider the Venn diagram as a way to show hierarchy of categories.

- How does the Venn diagram show the relationship between a category and a sub-category?
- To what other categories does a rhombus belong?
- How can using a Venn diagram help you make a tree diagram?

**For the tree diagram model**, prompt students to relate the tree diagram to the Venn diagram.

- How do categories of shapes in the tree diagram relate to categories in the Venn diagram?
- A trapezoid in the Venn diagram would be placed inside quadrilaterals, which is inside polygons. How is this shown in the tree diagram?

**Deepen Understanding**

**Tree Diagram Model**

**SMP 7** Use structure.

When discussing the tree diagram, prompt students to consider how the structure of a tree diagram allows specific shapes to fit in more than one category.

**Ask** How does the category that is farthest to the right relate to the two categories on its left?

**Listen for** The tree diagram moves from most general to most specific. This means the category farthest to the right, squares, is a sub-category of the categories it connects to, rectangles and rhombuses.

**Ask** When a shape is classified in a specific location on the tree diagram, how can you follow the branches to find all of the categories to which the shape belongs?

**Listen for** Because the categories move from most general to most specific, you can follow branches backwards from a shape's classification to more general categories. For example, in the tree diagram you can move from trapezoid to quadrilateral to polygon. When a shape is classified as specifically as possible, you can't classify it in any of the categories to the right on the tree diagram.
Ask problem 5. Listen for the idea that level farthest to the right or at the bottom of the tree diagram is the most specific classification and has the properties of all the categories above it.

Support Whole Class Discussion

Tell students that these problems will prepare them to provide the explanation required in problem 5.

Ask  Where does shape A belong in the tree diagram? How can you decide what other categories shape A or a square can be classified as?

Listen for The most specific category for shape A is a rhombus. Shapes belong to all of the more general categories that are directly connected when you move left (or up) in a tree diagram. Because square is directly connected on the left to both rectangle and rhombus, a square can be classified as both a rectangle and a rhombus, as well as all of the other categories to the left of rectangle and rhombus.

Look for the idea that level farthest to the right or at the bottom of the tree diagram is the most specific classification and has the properties of all the categories above it.

Reflect Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to use tree diagrams to classify two-dimensional figures.

1. Look at the Venn Diagram. Which part of the diagram shows the most general category? Which part of the diagram shows the most specific category?
   
   Possible answer: The outer rectangle shows the most general category - polygons. The sections that have no sub-categories inside them are the most specific categories – trapezoids and squares.

2. How is the tree diagram similar to the Venn diagram? How is it different?
   
   Possible answer: Both diagrams show the hierarchy of the shapes, from most general to most specific. The Venn diagram shows this by regions, and the tree diagram shows this by branches.

3. How can you see all of the categories that a shape belongs to in a tree diagram?
   
   What are all of the categories that shape A belongs to?
   
   Possible answer: Each shape belongs to all of the categories that come before it, so follow the branch to the left to see all of the categories. Shape A is a rhombus. It is also a rectangle, parallelogram, quadrilateral, and polygon.

4. Why do the branches that come off Rectangles and Rhombuses both lead to Squares?
   
   Possible answer: A square belongs in both of these categories. A square is a rectangle because it is a quadrilateral with four right angles. A square is a rhombus because it is a quadrilateral with four sides that have the same length.

5. Explain the relationship between the properties of categories when you move left or right (or up or down) in a tree diagram.
   
   Possible answer: All of the properties in the level to the left (above) are also true of the level you are on. When you move to the right (downward) in a tree diagram, you are moving to a category that has all of the properties of the previous category, with additional properties.


   Some students may say they like using a tree diagram because it shows how categories of shapes are related in a linear fashion. Other students may prefer a Venn diagram because it shows which categories are contained within other categories.

Hands-On Activity

Sort objects into specific categories.

If . . . students are unsure about levels within a tree diagram, Then . . . use this activity to help them understand that each level leads to objects with more specific properties.

Materials For each group: set of 10 sticky notes with the following words: peppers, carrots, vegetables, potatoes, red pepper, hot peppers, seeded vegetables, green peppers, root vegetables, and sweet potatoes; large sheet of paper; markers

• Distribute prepared sticky notes, papers, and markers to the groups.
• Ask: What is true of all of the words on the sticky notes? [Possible answer: They are all vegetables.] This is the top level of your tree diagram.
• Have students create a tree diagram with the sticky notes, with branches coming off of “Vegetables.”
• Emphasize that each vegetable at the bottom of the tree diagram belongs to all of the categories it is connected to above.
**APPLY IT**

For all problems, encourage students to think about and possibly list the properties of the shapes as they work with the tree diagrams.

Problem 7 is an opportunity to draw students’ attention to the hierarchy of triangles when classified by side lengths. An isosceles triangle is defined as having at least two sides of equal length. An equilateral triangle has all three sides equal length, making it a subcategory of isosceles triangles.

7 Cyrus is not correct; See possible explanation on Student Worktext page.

8 See the completed tree diagram on the Student Worktext page.

**Close: Exit Ticket**

9 equilateral, isosceles, equiangular, acute, triangle; See possible explanation on the Student Worktext page.

Students’ solutions should indicate understanding of:
- the properties of an equilateral triangle
- the properties of an acute triangle
- the properties of an isosceles triangle
- an understanding of the hierarchy shown in a tree diagram

**Error Alert** If students’ solution is only equilateral, then review hierarchy of shapes in a tree diagram. Have students explain why the equilateral triangle not only belongs in the category equilateral triangles, but also in all categories above to which it is connected.

---

**Lesson 29 Classify Two-Dimensional Figures**

**APPLY IT**

Use what you just learned to solve these problems.

7 Cyrus made the following tree diagram to show the relationship among scalene triangles, equilateral triangles, and isosceles triangles. Is his diagram correct? Explain.

Possible answer: Cyrus’s diagram is not correct. His tree diagram shows that all isosceles triangles are equilateral, but this is not true. The diagram would be correct if Equilateral and Isosceles switch places.

8 An equilateral triangle has three sides that have the same length. An equilateral triangle may also be called an equiangular triangle because each of the three angles has the same measure, 60°. Insert an additional category in the tree diagram for equiangular triangles. Explain.

Equiangular triangles have three angles that each measure 60°. This makes them a subcategory of acute triangles.

9 Look at the tree diagrams in the two problems above. Shape E has all sides the same length. What are all categories of triangles to which this triangle belongs? Explain.

equilateral, isosceles, equiangular, acute, triangle; Possible explanation: Shape E has three equal sides, making it equilateral and three equal angles, making it equiangular. All of the classifications moving upward following lines in a tree diagram are also true of a given shape.
See possible statements on the student page. Students need only provide one statement. Students may also state that rhombuses and squares are both parallelograms or are both quadrilaterals.

Basic

1. Look at the tree diagram above. Write a statement about the relationship between rhombuses and squares.

Possible answer: All squares are rhombuses. A rhombus can be a square if all four angles are right angles. Not all rhombuses are squares.

### Fluency & Skills Practice

**Assign Classifying Two-Dimensional Figures with Tree Diagrams**

In this activity students practice making a tree diagram and using it to classify shapes. Understanding how categories are related and how to classify shapes into more than one category based on the properties will develop students’ analytical thinking as well as their understanding of the categories and subcategories of geometric figures.
Lesson 29  Classify Two-Dimensional Figures

2. Yes; See possible explanation on student page. Students should recognize in a tree diagram when two lines lead to the same category, this category contains all of the properties of the two categories that connect to it.

Medium

3. See the tree diagram on the student page.

Medium

4. See possible explanation on the student page.

Medium

5. A (Always);
   E (Sometimes);
   I (Never);
   K (Sometimes)

Challenge

2. Look at the tree diagram on the previous page. Can a rhombus ever be a rectangle? Explain.

Yes; Possible explanation: A rhombus is a parallelogram with four equal length sides. A rectangle is a parallelogram with four right angles. A figure that has both four equal length sides and four right angles is a square, which is both a rhombus and a rectangle.

3. Draw a tree diagram to show the relationship among the following categories: polygons, pentagons, quadrilaterals, parallelograms, trapezoids, rectangles, and squares. Use the inclusive definition of trapezoid: a quadrilateral with at least one pair of parallel sides.

4. How would your tree diagram in the previous problem be different if you used the exclusive definition for trapezoid? Explain.

Possible answer: The exclusive definition of trapezoid states there are exactly two parallel sides. Quadrilaterals would branch into Trapezoids and Parallelograms. Nothing would be below Trapezoids, and Rectangles and Squares would be below Parallelograms.

5. Determine whether each statement is always, sometimes, or never true. Use the inclusive definition for trapezoid.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Always</th>
<th>Sometimes</th>
<th>Never</th>
</tr>
</thead>
<tbody>
<tr>
<td>A square is a parallelogram.</td>
<td>☐</td>
<td>☒</td>
<td>☑</td>
</tr>
<tr>
<td>A trapezoid is a rectangle</td>
<td>☐</td>
<td>☐</td>
<td>☒</td>
</tr>
<tr>
<td>A pentagon is a parallelogram.</td>
<td>☐</td>
<td>☐</td>
<td>☒</td>
</tr>
<tr>
<td>A trapezoid is a square.</td>
<td>☐</td>
<td>☐</td>
<td>☒</td>
</tr>
</tbody>
</table>

Prepare for Session 4

Use with Apply It.

Levels 1–3

**Listening/Speaking** Pair students. Read Apply It problem 1 aloud. Have students circle the terms quadrilaterals, polygons, trapezoids, and hexagons. Encourage students to create a table to describe the properties of each shape. Have students write the name and draw a picture of each shape on an index card, and manipulate the cards to arrange their diagrams.

Levels 2–4

**Speaking/Writing** Pair students. Read Apply It problem 1 aloud. Encourage partners to create a table of the shapes and their properties to plan their diagrams. Provide a bank of terms for students to use as they engage in conversations about classifying shapes. Possible terms may include classify, property, attribute, relationship, general, specific, parallel, and sides.

Levels 3–5

**Speaking/Writing** Have students form pairs and read Apply It problem 1. Ask students to discuss the shapes listed in the problem with their partner and how the categories of shapes are related. After they draw their Venn diagrams or tree diagrams, have students work together to think of phrases that tell about the relationship between quadrilaterals and trapezoids. Then ask them to write a sentence that describes the relationship. Have students share sentences with another set of partners and review.
**LESSON 29**

**SESSION 4**  
**Refine**

### Purpose
In this session students solve problems involving making and interpreting Venn diagrams or tree diagrams and then discuss and confirm their answers with a partner.

**Before students begin to work,** use their responses to the Check for Understanding to determine those who will benefit from additional support.

**As students complete the Example and problems 1–3,** observe and monitor their reasoning to identify groupings for differentiated instruction.

### Start

**Check for Understanding**

**Why** Confirm understanding of using Venn diagrams or tree diagrams to classify two-dimensional figures.

**How** Have students make a Venn diagram or a tree diagram using any strategy they want.

**Solution**

- Draw a Venn diagram or a tree diagram to classify these shapes.
- **Parallelograms**  
  **Polygons**  
  **Rhombuses**  
  **Quadrilaterals**

**Apply It**

1. **Draw a Venn diagram or a tree diagram to show the relationships among quadrilaterals, polygons, trapezoids, and hexagons.** Then write a statement about the relationship between quadrilaterals and trapezoids. **Show your work.**

   **Possible student work:**
   
   - **Trapezoids have all the attributes of quadrilaterals.**
   - **Hexagons**
   - **Quadrilaterals**
   - **Polygons**

**Pair/Share**

- **How can this tree diagram be shown with a Venn diagram?**
- **What is the most general category in your diagram?**
- **What does your diagram show about hexagons and quadrilaterals?**

### Error Alert

**If the error is . . .**

- any order other than the correct order

**Students may . . .**

- not be able to identify the properties of the given shapes.

**To support understanding . . .**

- Have students make a table with the names of the shapes as the headings of four columns. Work with students to list the properties of each of the shapes so that students can see, for example, that a rhombus has all the properties that parallelograms have plus one more.
**EXAMPLE**

The drawing of a square shown is one way to solve the problem. Students could also draw a rhombus.

**Look for** A square or a rhombus is a shape that has four sides and all equal sides.

**APPLY IT**

1. **Draw a Venn diagram or tree diagram to show the relationships among these polygons.**

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrilateral</td>
<td>polygon with exactly 4 sides</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>quadrilateral with exactly 1 pair of parallel sides</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>quadrilateral with 2 pairs of parallel sides</td>
</tr>
</tbody>
</table>

Possible diagram:

- **Quadrilaterals**
  - Parallelograms
  - Trapezoids

2. **Look at the Venn diagram below.**

Which statement is true?

- **A** An equilateral triangle is always an isosceles triangle.
- **B** An isosceles triangle is always an equilateral triangle.
- **C** Scalene and isosceles triangles share no attributes.
- **D** The label inside the largest oval could be Acute Triangles.

Brad chose **B** as the correct answer. How did he get that answer?

Possible answer: Brad thinks an isosceles triangle has all of the attributes of an equilateral triangle.

**PAIR/SHARE**

How would the diagram change if the description of the trapezoid used the definition quadrilateral with at least one pair of parallel sides?

**PAIR/SHARE**

What would you say to Brad to help him understand his mistake?

**APPLY IT**

2. **See the possible Venn diagram on the Student Worktext page. Students use the descriptions in the table to identify the relationships between the categories and make a Venn diagram or a tree diagram.**

**DOK 2**

**Look for** Polygons are the most general category in the diagram because polygons share with the other shapes only the attribute of being a closed shape with 3 or more straight sides.

3. **See the possible Venn diagram on the Student Worktext page. Students use the descriptions in the table to identify the relationships between the categories and make a Venn diagram or a tree diagram.**

**DOK 2**

**Look for** If a trapezoid is defined as having exactly one pair of parallel sides, the oval for trapezoids does not overlap with the oval for parallelograms because parallelograms have two pairs of parallel sides.

3. **A** Students could solve the problem by comparing each statement to the Venn diagram, noting that statement A is true because the oval for equilateral triangles is completely inside the oval for isosceles triangles.

Explain why the other two answer choices are not correct:

- **C** is not correct because all triangles share at least one property: having 3 sides and 3 angles.
- **D** is not correct because not all triangles have only acute angles. Some triangles also have right or obtuse angles.

**DOK 3**
4. Draw a tree diagram to show the relationships among triangles, quadrilaterals, isosceles triangles, and polygons.

Possible diagram:

- Polygons
  - Quadrilaterals
  - Triangles
  - Isosceles Triangles

5. Use the diagram in problem 4. Write two different statements that describe relationships among the shapes.

Answers will vary. Possible answer: Isosceles triangles have all the properties that triangles have. All triangles and quadrilaterals are polygons.

6. Could you add the two shapes below to your diagram in problem 4? If so, where would you put them? Name each shape as you explain your thinking.

Possible answer: You could add the pentagon, but not the circle. In the tree diagram, you could add a new branch below polygons, beside quadrilaterals and triangles. You could not add the circle to the tree diagram because a circle is not a polygon.

Error Alert  Students may think that a circle is a polygon with one side, so it belongs as a separate subcategory of polygons. Review the definition of polygon, a closed figure with 3 or more straight sides.

Differentiated Instruction

RETEACH

Hands-On Activity

Classify triangles using a Venn diagram and geoboards.

Students struggling with using Venn diagrams to sort and classify Will benefit from additional work with sorting shapes into Venn diagrams

Materials For each pair: geoboard, rubber bands; For display: Venn diagram with ovals for Triangles, Acute Triangles, and Isosceles Triangles (large enough for geoboards to be placed in each region; regions for acute and isosceles overlap)

- Have each pair make a triangle with their geoboard and rubber bands.
- Talk through the placement of the geoboard triangles in the Venn diagram. Ask: Does your geoboard belong inside the Triangles oval? inside the Acute Triangles oval? inside the Isosceles Triangles oval? Why? Listen for understanding of the properties of each pair’s triangle and how its properties determine its placement.
- For any region without a triangle in it, ask one pair to make a triangle that belongs in that region. Have them explain why the triangle belongs in the region.
- Change the label Acute Triangles to Right Triangles and repeat the activity.

EXTEND

Challenge Activity

Look at properties of kites.

Students who have achieved proficiency Will benefit from deepening understanding of classifying shapes

- Show students a shape that has two pairs of adjacent sides that are equal, one pair shorter than the other. Tell students that the shape is a kite.
- Have students list properties of the kite. [4 sides, 2 pairs of equal sides, opposite sides are not equal]
- Have students explain whether the kite would go inside or outside an oval for parallelograms in a Venn diagram.
Part A
Saul’s Venn diagram is not correct; See the possible explanation on the Student Worktext page. Students may draw a rhombus and a square to help solve the problem. Students may also say that the Venn diagram should show the oval for Squares completely inside the oval for Rhombuses because squares have all the properties of rhombuses.

Part B
See possible answer on the Student Worktext page; responses should show understanding that all squares can also be classified as rhombuses.

DOK 3

Part A
Is Saul’s Venn diagram correct? If not, what mistake did he make?
No; Possible explanation: Two pairs of parallel sides is also a property of squares, so this property should point to the region where the ovals overlap. The oval for squares should be completely inside the oval for rhombuses.

Part B
Describe the relationship between rhombuses and squares.
Squares are subcategory of rhombuses because a square has all of the properties of a rhombus. A square also has 4 right angles. Not all rhombuses have 4 right angles.

Math Journal
A regular polygon has all sides of equal length. Sami says that all squares, equilateral triangles, and pentagons can be classified as regular polygons. Is Sami correct? Draw a Venn diagram and explain your thinking.

Possible diagram:

No; Possible explanation: Squares and equilateral triangles are regular polygons because their sides all have equal length. A pentagon is any 5-sided polygon. Only some pentagons are regular pentagons.

Self Check
Go back to the Unit 4 Opener and see what you can check off.

Reinforce
Problems 4–8
Classify two-dimensional figures.

All students will benefit from additional work with classifying two-dimensional figures by solving problems in a variety of formats.

• Have students work on their own or with a partner to solve the problems.
• Encourage students to show their work.

Personalize

i-Ready

Provide students with opportunities to work on their personalized instruction path with i-Ready Online Instruction to:
• fill prerequisite gaps
• build up grade-level skills

Close: Exit Ticket

Math Journal
Student responses should indicate understanding that squares and equilateral triangles can be classified as regular polygons, but not all pentagons are regular; students may draw an example of a pentagon for which not all sides are the same length.

Error Alert If students say Sami is correct, then ask them to define pentagon, square, and equilateral triangle. Note that a pentagon is a closed shape with five sides. Draw a regular pentagon and a pentagon with at least two sides of different lengths. Ask if both are still pentagons.

Self Check Have students consider whether they feel they are ready to check off any new skills on the Unit 4 Opener page.
LESSON 29 Lesson Quiz

Tested Skills
Assesses 5.G.B.3, 5.G.B.4
Problems on this assessment form require students to be able to categorize two-dimensional figures based on properties and to interpret and Venn diagrams and tree diagrams to classify figures. Students will also need to be familiar with acute, obtuse, and right angles, parallel and perpendicular lines, equilateral and isosceles triangles, and how to sort figures based on side lengths and angle types. Alternately, teachers may assign the Digital Comprehension Check online to assess student understanding of this material.

Error Alert Students may:
• confuse, incorrectly identify, or ignore some attributes of different figures.
• assume a relationship in one direction in a tree diagram is also true in the other direction.
• not be able to interpret or complete a tree diagram.
• not be able to interpret a Venn diagram.
• not realize that at least one also includes more than one.

Solve the problems.
Use the tree diagram below to answer questions 1 – 3.

1 Which statement describes all rhombuses? (1 point)
   a polygon with three sides
   b a polygon with exactly 1 pair of parallel sides
   c a quadrilateral with 4 sides of equal length
   d a parallelogram with 4 right angles.

2 Alex wants to add this shape to the tree diagram using the exclusive definition of trapezoid.
Where should he place the shape on the tree diagram? Explain. (2 points)
Possible answer: The trapezoid is a quadrilateral because it has 4 sides. It is not a parallelogram because it has exactly 1 pair of parallel sides, not 2 pairs of parallel sides. Therefore, the trapezoid goes below quadrilaterals and to the side of parallelograms on the tree diagram.

3 Which category on the tree diagram is the least specific? Explain. (2 points)
Possible answer: Polygons are the least specific because all the other subcategories of shapes have all the attributes of a polygon.

Short Response Scoring Rubric

<table>
<thead>
<tr>
<th>Points</th>
<th>Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Response contains the following:</td>
</tr>
<tr>
<td></td>
<td>• Correct solutions and/or reasoning. (1 point)</td>
</tr>
<tr>
<td></td>
<td>• Well-organized, clear, and concise work that demonstrates thorough understanding of math concepts. (1 point)</td>
</tr>
<tr>
<td>1</td>
<td>Response contains the following:</td>
</tr>
<tr>
<td></td>
<td>• Mostly correct solution(s).</td>
</tr>
<tr>
<td></td>
<td>• Shows partial or good understanding of math concepts.</td>
</tr>
<tr>
<td>0</td>
<td>Response contains the following:</td>
</tr>
<tr>
<td></td>
<td>• Incorrect solution(s).</td>
</tr>
<tr>
<td></td>
<td>• No attempt at finding a solution.</td>
</tr>
<tr>
<td></td>
<td>• No effort to demonstrate an understanding of mathematical concepts.</td>
</tr>
</tbody>
</table>

Choice Matrix Scoring Rubric

<table>
<thead>
<tr>
<th>Points</th>
<th>Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 points</td>
<td>All answers are correct</td>
</tr>
<tr>
<td>1 point</td>
<td>1 incorrect answer</td>
</tr>
<tr>
<td>0 points</td>
<td>2 or more incorrect answers</td>
</tr>
</tbody>
</table>
Solutions

1 C; Students could solve the problem by using what they know about properties to identify rhombuses as a subcategory of quadrilaterals and parallelograms with 4 sides of equal length. A is not correct because rhombuses are polygons with 4 sides, not 3 sides. B is not correct because rhombuses have 2 pairs of parallel sides, not 1 pair of parallel sides. D is not correct because rhombuses are parallelograms with opposite equal angles, but may not have 4 right angles. 1 point 5.G.B.3, DOK 2

2 See possible answer on the student page. 2 points 5.G.B.4, DOK 2

3 See possible answer on the student page. 2 points 5.G.B.3, DOK 2

4 C (Never); E (Sometimes); G (Always); K (Sometimes); O (Never) 2 points 5.G.B.3, DOK 2

5 See completed classification on the student page. 2 points 5.G.B.4, DOK 2

Differentiated Instruction

RETEACH

Tools for Instruction

Students who require additional support for prerequisite or on-level skills

Will benefit from activities that provide targeted skills instruction

REINFORCE

Math Center Activities

Students who require additional practice to reinforce concepts and skills and deepen understanding

Will benefit from small group collaborative games and activities (available in three versions—on-level, below-level, and above-level)

EXTEND

Enrichment Activities

Students who have achieved proficiency with concepts and skills and are ready for additional challenges

Will benefit from group collaborative games and activities that extend understanding